Technical Note

Kitagawa-Takahashi diagrams define the limiting conditions for cyclic fatigue failure in human dentin

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Abstract: As cyclic fatigue is considered to be a major cause of clinical tooth fractures, achieving a comprehensive understanding of the fatigue behavior of dentin is of importance. In this note, the fatigue behavior of human dentin is examined in the context of the Kitagawa-Takahashi diagram to define the limiting conditions for fatigue failure. Specifically, this approach incorporates two limiting threshold criteria for fatigue: (i) a threshold stress for fatigue failure, specifically the smooth-bar (unnotched) fatigue endurance strength, at small crack sizes and (ii) a threshold stress-intensity range for fatigue-crack growth at larger crack sizes. The approach provides a "bridge" between the traditional fatigue life and fracture mechanics based damage-tolerant approaches to fatigue-life estimation, and as such defines a "failure envelope" of applied stresses and flaw sizes where fatigue failure is likely in dentin This approach may also be applied to fatigue failure in human cortical bone (i.e. clinical "stress fractures"), which exhibits similar fatigue behavior characteristics, and in principle may aid clinicians in making quantitative evaluations of the risk of fractures in mineralized tissues. © 2006 Wiley Periodicals, Inc. J Biomed Mater Res 79A: 747–751, 2006

Key words: dentin; bone; mineralized tissue; fatigue-crack growth threshold; fatigue life

INTRODUCTION

The importance of the fatigue behavior of mineralized tissues was recognized roughly half a century ago in the work of Evans and Lebow on human cortical bone[1]; indeed, clinical stress fractures in human bone are believed to result from continued repetitive (cyclic fatigue) loading, rather than a single traumatic loading event.[2,3] Cyclic stresses play a similarly important role in the failure of teeth. Early studies suggested that thermal cycling could generate the cyclic stresses causing fatigue damage in teeth,[4] although most recent work has focused on the more important role of mechanical fatigue damage, specifically from masticatory stresses.[5–10]

Two basic methodologies are commonly used to deal with the problem of fatigue in materials. The first of these is the traditional fatigue life (or *S*/*N*) approach, where unnotched, nominally flaw-free, specimens are cycled at various stress or strain levels (*S*) in order to determine the number of cycles to failure (*N*), specifically both to initiate and propagate a crack to failure. The second is the more recently developed damage-tolerant approach where fatigue-crack propagation is characterized by the crack-growth rate per cycle, d*a*/d*N*, as a function of the linear-elastic stress-intensity range, $\Delta K = K_{max} - K_{min}$,* where K_{max} and K_{min} are the maximum and minimum stress intensities during the loading cycle.[12] In this latter case, the fatigue life may be predicted from crack-growth data by determining the number

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*The stress-intensity factor, *K*, is a global parameter, which fully characterizes the local stress and deformation fields in the immediate vicinity of a crack tip in a linear-elastic solid, and thus can be used to correlate to the extent of crack advance. It is defined for a crack of length *a* as $K = Y\sigma_{app}(\pi a)^{1/2}$, where σ_{app} is the applied stress and *Y* is a geometry factor of order unity.[11]

of cycles to grow a pre-existing flaw to the size required to cause final fracture of the material.

From a perspective of quantitatively assessing fatigue damage, the S/N approach is easier to apply experimentally, and consequently has received the most attention for characterizing the fatigue behavior of mineralized tissues, such as teeth and bone.[1,5,7,13–20] However, the data can often be marred by excessive scatter because of variations in surface condition or flaw distributions,[21] which can have a large effect on the crack initiation portion of the fatigue life. Additionally, from a perspective of identifying fatigue mechanisms, it is difficult to separate factors that govern the crack initiation and growth stages of the fatigue lifetime using total life S/N studies. In contrast, the fracture mechanics based damage-tolerant approach focuses solely on crack growth, and as such provides a more accurate way to characterize fatigue behavior in the presence of pre-existing flaws. Since microcrack damage is often a characteristic of mineralized tissues such as dentin and bone, this approach has received considerably more attention of late for evaluating the fatigue resistance of these materials.[5,6,8–10,22–24] Unfortunately, it is difficult to apply this approach where crack sizes become small when compared with the extent of local plasticity, or more importantly, the scale of microstructure, *e.g.*, the size and spacing of the tubules in dentin ($\sim$micrometers) or osteons ($\sim$hundreds of micrometers) in cortical bone. As these crack sizes are not atypical in dentin and bone, it would be pertinent to utilize a fatigue assessment method which marries these two approaches and generates a description of how the limiting stress to cause fatigue failure varies with pre-existing crack size. Such an approach is afforded by the Kitagawa-Takahashi diagram.[25]

Accordingly, in this note a Kitagawa-Takahashi diagram for the fatigue failure of human dentin is presented. It is believed that a similar diagram could also be readily constructed for human cortical bone if appropriate data were available, and that this methodology could provide clinicians with a basis for new and improved tools for predicting fracture risk.

PROCEDURES

To construct the Kitagawa-Takahashi diagram, recently published data on the fatigue behavior of human dentin is used,[5] where both S/N data and $da/dN-\Delta K$ data were collected from the same donor set of recently extracted human molars. Twenty-five beams of γ radiation sterilized dentin ($0.9 \times 0.9 \times 10$ mm^3) were used in that study. Because of the natural curvature of the dentinal tubules through the dentin, their orientation was not consistent at all positions along the beam. However, post-test analy-

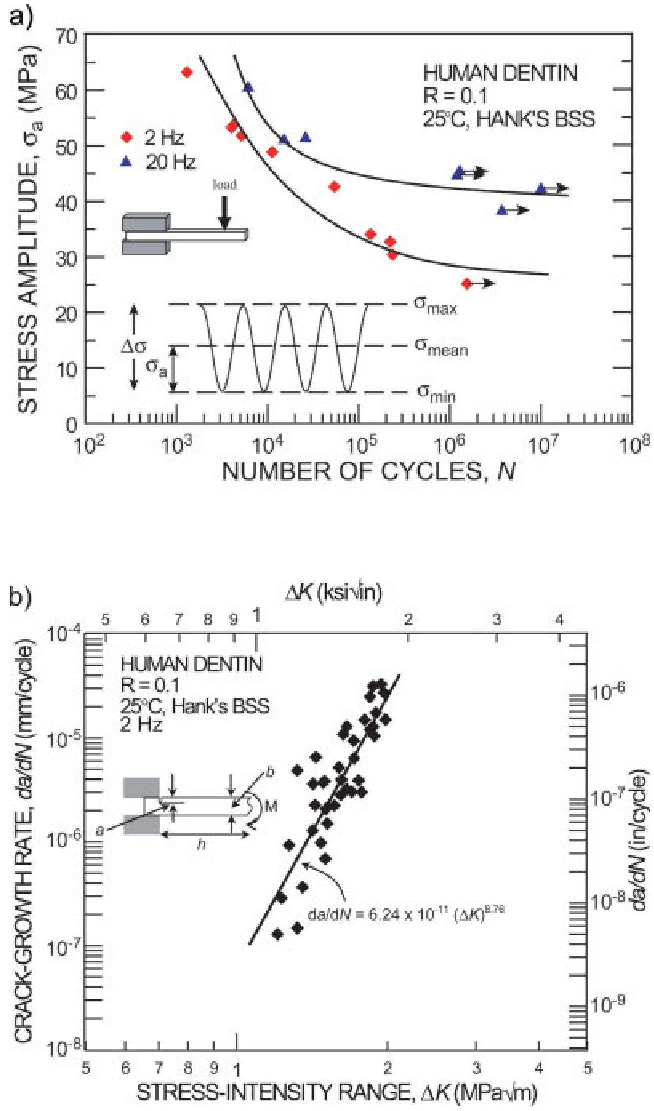

Figure 1. (a) Traditional stress-life (S/N) fatigue data and (b) damage-tolerant fatigue-crack growth ($da/dN - \Delta K$) data for human dentin, tested in 25°C Hanks' balanced salt solution, taken from Ref. 5. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

sis of fracture surfaces indicated that the fracture plane was oriented perpendicular to the average tubule direction, with the tubules running generally along the long axis of the beam locally at the point of fracture. Quasi-static strength properties in Hanks' balanced salt solution (HBSS) gave yield and flexural strengths of $\sigma_y = 75$ MPa and $\sigma_F = 160$ MPa, respectively. *Ex vivo* S/N fatigue tests were conducted on unnotched cantilever beams [Fig. 1(a) inset], tested in 25°C HBSS with a load ratio (minimum load/maximum load) of $R = 0.1$ at cyclic frequencies of 2 and 20 Hz; results are shown in Figure 1(a) in terms of the stress amplitude, σ_a, vs. the number of cycles to failure, N_f. Fatigue-crack growth data were additionally computed from these experiments using

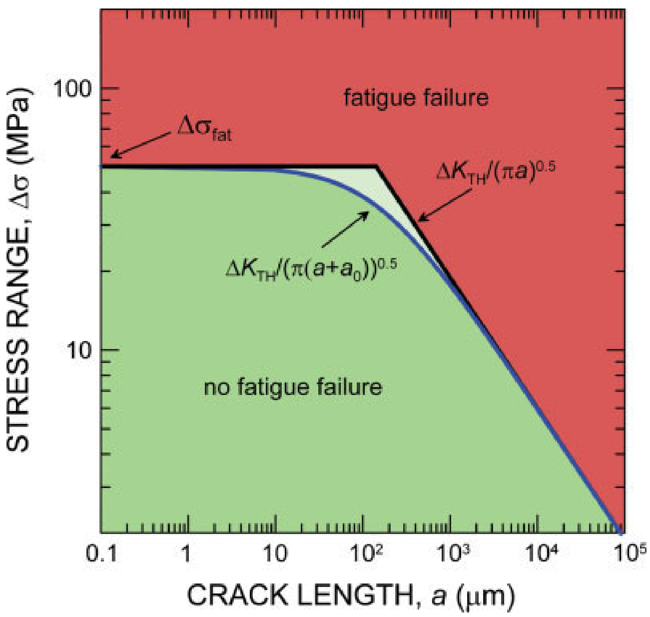

Figure 2. The Kitagawa-Takahashi diagram for human dentin, constructed from the data in Figure 1, which defines the stress range for fatigue failure (within $\sim 10^6$ cycles) as a function of crack size. The diagram is based on the limiting conditions for fatigue failure, which comprise the fatigue endurance strength ($\Delta\sigma_{fat}$) at small crack sizes, and the fatigue threshold stress-intensity range (ΔK_{TH}) at larger crack sizes. The transition between these limiting conditions is described by the traditional [Eq. (1)] and El Haddad et al.[26] [Eqs. (2,3)] approaches. This diagram defines a "failure envelope" of crack size/stress range combinations for fatigue failure in teeth. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

measurements of the loss in stiffness of the cantilever-beam specimens throughout the fatigue test; these growth-rate results are shown in Figure 1(b) as a function of the stress-intensity range. Details of the procedures used to generate the results in Figure 1 may be found in Ref. 5.

To construct the Kitagawa-Takahashi diagram from the data in Figure 1, the limiting conditions for fatigue failure must be considered in the presence of small and large crack sizes. The unnotched cantilever-beam *S/N* data provide information for small crack sizes; specifically the limiting condition is afforded by the fatigue endurance strength at greater than 10^6 cycles. Using the results in Figure 1(a) for cycling at 2 Hz, which are more representative of actual *in vivo* loading conditions, the endurance strength for dentin (at $R = 0.1$) can be seen to be $\Delta\sigma_{fat} \sim 50$ MPa ($\sigma_a \sim 25$ MPa). Note, the stress range, $\Delta\sigma = \sigma_{max} - \sigma_{min}$, is used for the Kitagawa-Takahashi diagram rather than the stress amplitude, $\sigma_a = ½(\sigma_{max} - \sigma_{min})$, which is commonly plotted for *S/N* data, to match the corresponding limiting condition at large crack sizes, which is given by the threshold stress-intensity range for fatigue-crack growth, ΔK_{TH}. This value represents the stress-intensity range below which cracks appear dormant, operationally defined here when $da/dN < 10^{-7}$ mm/cycle. From the data in Figure 1(b), the fatigue threshold for dentin (at $R = 0.1$) can be seen to be $\Delta K_{TH} \sim 1.06$ MPa$\sqrt{m}$.

The premise of the Kitagawa-Takahashi method is that the stress-intensity function can be plotted for the fatigue threshold, ΔK_{TH}, in terms of the stress range, $\Delta\sigma$, and the crack size, a, viz:

$$\Delta\sigma = \frac{\Delta K_{TH}}{Y\sqrt{\pi a}} \tag{1}$$

Although the value of ΔK_{TH} is nominally independent of crack size, at small crack sizes this function tends to a limiting condition given by the fatigue endurance strength, which for dentin is $\Delta\sigma_{fat} \sim 50$ MPa. Defining a_0 as the crack length below which the fatigue threshold is no longer crack size independent, then the variation in stress range for fatigue can be expressed as a function of crack length, a, by El Haddad et al.'s single empirical equation:[26]

$$\Delta\sigma = \frac{\Delta K_{TH}}{Y\sqrt{\pi(a + a_0)}} \tag{2}$$

which captures both small- and large-crack limiting conditions by introducing a new length scale a_0. This length scale, sometimes referred to as the intrinsic crack size,[26] can be calculated from:

$$a_0 = \left(\frac{\Delta K_{TH}}{Y\Delta\sigma_{fat}}\right)^2 \frac{1}{\pi} \tag{3}$$

This intrinsic crack size is on the order of $a_0 \sim 150$ μm for human dentin.

Of the two approaches given in Eqs. (1)–(3), the El Haddad et al. method[26] [Eqs. (2,3)] is slightly more conservative, although both methods are applied in the present work. For simplicity, the geometry factor Y was chosen to be unity, since specific flaw geometry was not analyzed. Values of Y for specific crack configurations and loading conditions can be found in standard handbooks.[27,28]

RESULTS AND DISCUSSION

On the basis of measured *S/N* and fatigue threshold data in Ref. 5, the Kitagawa-Takahashi diagram for human dentin was determined and is shown in Figure 2; both the standard [Eq. (1)] and El Haddad et al. [Eqs. (2,3)][26] constructions are plotted. This diagram defines the variation in stress range to cause fatigue failure in dentin as a function of crack size.

What is important about this diagram is that it indicates that the limiting stress for fatigue is not given by a single constant parameter, such as the fatigue endurance strength, as is implied by the traditional *S/N* approach, but rather that the limiting stress for fatigue failure varies markedly with preexisting flaw size. In this regard, the Kitagawa-Takahashi diagram provides the "failure envelope" of crack size/stress range combinations where fatigue failure (in the present case, within $\sim 10^6$ cycles[†]) may be a concern. The main strength of this approach is that it gives a straightforward and quantitative way to evaluate whether a known flaw will pose a failure risk under conditions of, for example, masticatory stresses in human teeth. It should be noted that the failure envelope in Figure 2 pertains to a load ratio of $R = 0.1$, a situation where the dentin is essentially unloaded on each cycle, as may be expected during normal chewing. However, for situations where other load ratios are of interest one must construct the Kitagawa-Takahashi diagram using fatigue data for the relevant load ratio; indeed, both the fatigue threshold and fatigue strength of dentin are known to be sensitive to changes in the load ratio.[7,9]

Also, because of the scatter which is prevalent in *S/N* data, one must be careful to ensure that an adequate data set is collected to determine the fatigue strength used in constructing the Kitagawa-Takahashi diagram. In the present case, limited available data was used because, in general, the effect of mechanical fatigue on dentin has received comparatively little detailed attention in the literature, at least until recently.[5–10] However, it is apparent that further advances in dentistry to prevent tooth fractures will require the application of modern methods for predicting fatigue failures. Furthermore, there are several applications where the Kitagawa-Takahashi approach could provide a quantitative means to assess the fracture risk associated with flaws in teeth provided adequate methods are available for detecting cracks at relevant size scales. One example is flaws introduced during reparative dental procedures. There are several reports that suggest that restored teeth are more likely to experience fracture over time.[29–31] In cases where such failures are initiated at introduced damage from the procedure, one should be able to quantitatively assess the failure risk using this approach. Another example is noncarious notches in dentin often observed on exposed root surfaces just below the enamel-cementum junction.[32,33] Such flaws are potential sites for the initiation of tooth failure,[34] and whereas fracture at these sites could in principle result from a single catastrophic loading event, it is generally believed that these failures are the end result of subcritical cracking induced by repetitive occlusal stresses, *i.e.,* from cyclic fatigue damage.[34] In these applications, the Kitagawa-Takahashi diagram, coupled with an assessment of flaw size, *e.g.,* from X-ray examination, may provide an instant quantitative assessment of the likelihood of premature tooth failure. Although traditional dental X-ray radiographs would be insufficient in resolution for such applications, state-of-the-art commercial X-ray computer tomography (CT) equipment developed recently for dental applications now achieve minimum resolutions of ~ 250 μm; accordingly, with slightly improved resolution this approach could find significant clinical use.

Application of the Kitagawa-Takahashi approach to cortical bone is less straightforward because of the role of fatigue damage in inducing remodeling and repair by basic multicellular units;[35–37] also complicating the situation is the ability of bones to change their geometry and mechanical properties to adapt to long-term changes in loading patterns.[2,38] Using such life-prediction methodologies, however, may still be instructive as they provide worst-case estimates of the life of bone in the presence of short term, high magnitude, cyclic loading. Indeed, with improvements of nondestructive techniques to evaluate flaws or damage in bone, high risk patients for stress fractures, such as athletes and military recruits, could be monitored and quantitative evaluation of fatigue fracture risk could be made based on the magnitude of any detected fatigue damage.

[†]Since this approximates to roughly 1 year or so, for longer term predictions, *S/N* data out to $\sim 10^7$–10^8 cycles would be required.

CONCLUSION

In summary, we have described how the Kitagawa-Takahashi diagram may be used for generating quantitative predictions of the conditions, in terms of applied stress and flaw size, for fatigue failure in human dentin. We believe that with this methodology, in combination with an evaluation of damage and/or flaw sizes, it may be possible to make quantitative evaluations of the risk of premature fatigue fractures in teeth; however, verification of the practical utility of this method must be made in future studies.

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