

Mechanisms of fatigue-crack propagation in ductile and brittle solids

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Abstract. The mechanisms of fatigue-crack propagation are examined with particular emphasis on the similarities and differences between cyclic crack growth in ductile materials, such as metals, and corresponding behavior in brittle materials, such as intermetallics and ceramics. This is achieved by considering the process of fatiguecrack growth as a mutual competition between *intrinsic* mechanisms of crack advance ahead of the crack tip (e.g., alternating crack-tip blunting and resharpening), which promote crack growth, and *extrinsic* mechanisms of crack-tip shielding behind the tip (e.g., crack closure and bridging), which impede it. The widely differing nature of these mechanisms in ductile and brittle materials and their specific dependence upon the alternating and maximum driving forces (e.g., ΔK and K_{max}) provide a useful distinction of the process of fatiguecrack growth and crack size. Finally, the differing susceptibility of ductile and brittle materials to cyclic degradation has broad implications for their potential structural application; this is briefly discussed with reference to lifetime prediction.

Key words: Fatigue-crack propagation, crack-tip shielding, metals, ceramics, intermetallics, intrinsic and extrinsic mechanisms.

1. Introduction

Cyclic fatigue involves the microstructural damage and failure of materials under cyclically varying loads. Structural materials, however, are rarely designed with compositions and microstructures optimized for fatigue resistance. Metallic alloys are generally designed for strength, intermetallics for ductility, and ceramics for toughness; yet, if any of these materials see engineering service, their structural integrity is often limited by their mechanical performance under cyclic loads. In fact, it is generally considered that over 80 percent of all service failures can be traced to mechanical fatigue, whether in association with cyclic plasticity, sliding or phsyical contact (fretting and rolling contact fatigue), environmental damage (corrosion fatigue), or elevated temperatures (creep fatigue). Accordingly, a large volume of literature has been amassed particularly over the past twenty-five years, dealing with the mechanics and mechanisms of mechanical fatigue failure [e.g., Suresh, 1991; Ellyin, 1997]; however, the vast majority of this research pertains solely to metallic materials.

Despite this preponderance of information on metal fatigue, there has been an increasing interest of late in the use of high-strength, brittle materials, such a ceramics, intermetallics and their respective composites, for structural applications where cyclic loading is critical [e.g., Harrison and Winstone, 1996; Kochendörfer, 1996]. This has been particularly focused at elevated temperature applications, e.g., for fuselage and especially engine components (Kochendörfer, 1996), but in the case of ceramics at lower temperatures too, e.g., for biomed-



Figure 1. Schematic variation of fatigue-crack propagation rate (da/dN) with applied stress intensity range (ΔK) , for metals, intermetallics and ceramics.

ical implant devices (Ritchie, 1996). Examples of such 'advanced materials' are the use of silicon nitride ceramics for automobile turbocharger wheels and engine valves and pyrolytic carbon for prosthetic cardiac devices, and the contemplated use of composite ceramics for gas turbine blades. Similarly, intermetallic alloys, such as the γ -based titanium aluminides, have been considered for applications such as automobile engine valves and blades in gas turbines. Whereas these materials offer vastly improved specific strength at high temperatures compared to conventional metallic alloys, they suffer in general from a pronounced lack of damage tolerance in the form of an extreme sensitivity to pre-existing flaws. Moreover, it has become apparent that similar to metals, such brittle solids can additionally show a marked susceptibility to premature failure under cyclic fatigue loading [e.g., Dauskardt et al., 1987].

The mechanisms associated with fatigue-crack propagation in brittle materials, such as monolothic and composite ceramics and intermetallics, are quite distinct from those commonly encountered in metal fatigue; moreover, their crack-growth rate (da/dN) behavior displays a markedly higher sensitivity to the applied stress intensity (*K*) than is observed in most metals (Figure 1) (Ritchie and Dauskardt, 1991). However, by considering crack growth as a mutual competition between *intrinsic* microstructural damage mechanisms, which promote crack extension *ahead* of the tip, and *extrinsic* crack-tip shielding mechanisms, which act primarily *behind* the tip to retard crack growth (Ritchie, 1988) a specific commonality of behavior between the fatigue of ductile and brittle materials can be found, differing only in the relative importance of the intrinsic and extrinsic mechanisms.

It is therefore the objective of this paper to describe this commonality by comparing and contrasting the salient mechanisms affecting the propagation of fatigue cracks in ductile and



Figure 2. Schematic illustration of mutual competition between intrinsic mechanisms of damage/crack advance and extrinsic mechanisms of crack-tip shielding involved in crack growth.

brittle materials. We begin with a brief review of the distinction between the intrinsic and extrinsic mechanisms which can affect the critical and subcritical growth of cracks.

2. Intrinsic and extrinsic mechanisms

As noted above, the critical and subcritical extension of a crack can be considered to be a result of the mutual competition of two classes of mechanisms (Figure 2). Crack growth is promoted *ahead* of the crack tip by *intrinsic* microstructural damage mechanisms, and impeded by *extrinsic* mechanisms acting primarily *behind* the crack tip, which serve to 'shield' the crack tip from the far-field driving forces (Ritchie, 1988).

In metallic materials, intrinsic damage mechanisms typically involve processes which create microcracks or voids, e.g., by dislocation pile-ups or interface decohesion, in the highly stressed region ahead of the tip, leading to classical failure by cleavage, intergranular cracking or microvoid coalescence; comparable mechanisms under cyclic loads involve the repetitive blunting and resharpening of the crack tip [e.g., Pelloux, 1969; Neumann, 1969]. Extrinsic shielding mechanims, conversely, result from the creation of inelastic zones surrounding the crack wake or from physical contact between the crack surfaces via wedging, bridging, sliding or combinations thereof (Figure 3) (Ritchie, 1988). Examples of 'zone shielding' are transformation and microcrack toughening in ceramics and rocks, where the *in situ* dilatant phase transformations or the microcracking of precipitates/particles ahead of the crack tip can lead to inelastic zones in the crack wake which impart closing tractions on the crack surfaces. Examples of 'contact shielding' are the bridging tractions imposed across a crack by unbroken fibers, laminated layers or a particulate phase in composite materials, or the wedging of corrosion debris or fracture surface asperities during crack closure in metal fatigue.

It is important to note that the intrinsic mechanisms are an inherent property of the material, and thus are active irrespective of the length of the crack or the geometry of the test specimen; under monotonic loads, for example, they control the driving forces (e.g., the stress intensity) to *initiate* cracking. Extrinsic mechanisms, conversely, act in the crack wake and are thus

EXTRINSIC TOUGHENING MECHANISMS



Figure 3. Schematic illustration of the mechanisms of extrinsic toughening, involving crack deflection and crack-tip shielding by inelastic zone or contact between the crack surfaces (Ritchie, 1988).

critically dependent on crack size and (to a lesser extent) geometry; they are responsible for the development of resistance-curve (R-curve) behavior and thus play a prominent role in the driving forces required for continued *growth* of the crack. The implications of this are that where extrinsic shielding mechanisms are active, rising R-curve toughness behavior and 'small-crack' effects are to be expected, both phenomena resulting from the crack-growth properties being dependent upon crack size. Moreover, since extrinsic mechanisms can have no effect on crack initiation (since there is no crack wake), the microstructural factors affecting (large) crack growth may be quite different from those affecting crack initiation (or small crack growth). In general, ductile materials are toughened intrinsically, e.g., through mobile dislocation activity to induce a significant plastic-zone size, although under cyclic loading extrinsic mechanisms play an important role in the form of crack closure. In contrast, brittle materials, such as ceramics, are invariably toughened extrinsically [e.g., Evans, 1990; Becher, 1991], via such mechanisms as transformation toughening and crack bridging, the latter through interlocking grains in many monolithic ceramics or by uncracked ligaments or unbroken reinforcement phases in composites and laminates.

From the perspective of finding any commonality in mechanisms of fatigue-crack growth in different materials, *it is the specific nature and, more significantly, the relative importance of the intrinsic (damage) versus extrinsic (shielding) mechanisms which distinguishes the cyclic fatigue behavior of ductile and brittle solids.* This is in turn governs the specific dependencies of the alternating and maximum stress intensities on crack-growth rates, i.e., how da/dNdepends upon ΔK and K_{max} (and thus how the resulting lifetime is a function of the alternating or maximum stresses), and the relationships between the thresholds for fatigue-crack growth $(\Delta K_{TH} \text{ and } K_{max,TH})$ and the crack-initiation (K_o) and steady-state (K_c) fracture toughness values.

We begin with a brief review of the mechanics and mechanisms affecting cyclic crack growth in ductile metallic materials.

3. Fatigue-crack propagation in ductile metallic materials

3.1. GENERAL CONSIDERATIONS

Subcritical crack growth can occur at stress intensity K levels generally far less than the fracture toughness K_c in any metallic alloy when cyclic loading is applied ($\Delta K_{TH}/K_c \sim 0.1 - 0.4$). In simplified concept, it is the accumulation of damage from the cyclic plastic deformation in the plastic zone at the crack tip that accounts for the intrinsic mechanism of fatigue crack growth at K levels below K_c . The process of fatigue failure itself consists of several distinct processes involving initial cyclic damage (cyclic hardening or softening), formation of an initial 'fatal' flaw (crack initiation), macroscopic propagation of this flaw (crack growth), and final catastrophic failure or instability.

The physical phenomenon of fatigue was first seriously considered in the mid nineteenth century when widespread failures of railway axles in Europe prompted Wöhler in Germany to conduct the first systematic investigations into material failure under cyclic stresses circa 1860 (Wöhler, 1860). However, the main impetus for research directed at the crack propagation stage of fatigue failure, as opposed to mere lifetime calculations, did not occur until the mid 1960s, when the concepts of linear elastic fracture mechanics and so-called 'defect-tolerant design' were first applied to the problem of subcritical flaw growth (Paris et al., 1961; Johnson and Paris, 1967). Such approaches recognize that all structures are flawed, and that cracks may initiate early in service life and propagate subcritically. Lifetime is then assessed on the basis of the time or number of loading cycles for the largest undetected crack to grow to failure, as might be defined by an allowable strain, or limit load, or K_c criterion. Implicit in such analyses is that subcritical crack growth can be characterized in terms of some governing parameter (often thought of as a crack driving force) that describes local conditions at the crack tip yet may be determined in terms of loading parameters, crack size, and geometry. Linear elastic and nonlinear elastic fracture mechanics have, to date, provided the most appropriate methodology for such analyses to be made.



Figure 4. Schematic illustration of the typical variation in fatigue-crack growth rates da/dN, as a function of the applied stress-intensity range ΔK in metallic materials, showing the regimes of primary growth-rate mechanisms and effects of several major variables on crack-growth behavior (Ritchie, 1977).

The general nature of fatigue-crack growth in metallic materials and its description using fracture mechanics can be briefly summarized by the schematic diagram in Figure 4 showing the variation in da/dN with the nominal stress-intensity range ($\Delta K = K_{max} - K_{min}$) (Ritchie, 1977). In actuality, the growth rates depend upon numerous factors other than ΔK , although this is the primary variable in metal fatigue, viz.:

$$da/dN = f[\Delta K, K_{\text{max}} \text{ (or } R), v, \text{ environment, wave form} \dots],$$
(1)

where the load ratio R is the ratio of minimum to maximum applied loads (= K_{\min}/K_{\max} for positive R), and v is the frequency. Specifically, results of fatigue-crack growth rate tests for most ductile materials display the following characteristics: (1) a region at low values of ΔK and da/dN (less than $\sim 10^{-9}$ m/cycle) in which fatigue cracks appears dormant below the fatigue threshold, ΔK_{TH} ; (2) an intermediate region ($\sim 10^{-9}$ to 10^{-6} m/cycle) of power-law behavior described by the Paris equation (Paris and Erdogan, 1963):

$$\mathrm{d}a/\mathrm{d}N = C(\Delta K)^m,\tag{2}$$

where *C* and *m* (~ 2 to 4) are material scaling constants; and (3) an upper region of accelerating crack growth (above ~ 10^{-6} m/cycle) as K_{max} approaches K_c or gross plastic deformation of the specimen, e.g., at the limit load. Similar approaches have been suggested for crack growth under large-scale yielding [e.g., Dowling and Begley, 1976] where growth rates have been related to a cyclic *J*-integral (ΔJ) or range of crack-tip opening displacement (Δ CTOD).



Figure 5. Fatigue-crack propagation in a bulk amorphous metal (metallic glass), $Zr_{41.2}Ti_{13.8}Cu_{12.5}Ni_{10}Be_{22.5}$, showing (a) fatigue-crack propagation rates scaling with the $\Delta CTOD = 0.01 \Delta K^2 / \sigma_0 E'$ (where σ_0 is the flow stress and E' is the appropriate Young's modulus), and (b) crack growth occurring via ductile striation formation (Gilbert et al. 1997). Arrow in (b) indicates direction of crack growth.

3.2. MECHANISTIC ASPECTS

3.2.1. Intrinsic mechanisms

Fatigue failure in metals is generally characterized by a transgranular ductile striation mechanism. Such striations represent *local* crack-growth increments per cycle and have been hypothesized to oocur via a mechanism of opening and blunting of the crack tip on loading, followed by resharpening of the tip on unloading (Laird and Smith, 1962). Several theoretical models for such growth (often termed stage II crack propagation) have been proposed that rely on the fact that, where plastic zones are sufficiently large compared to microstructural dimensions, plastic blunting at the crack tip is accommodated by shear on two slip-systems roughly 45° to the crack plane (Pelloux, 1969; Neumann, 1969). Recognizing that such sliding-off is largely irreversible, new crack surface can be created during cyclic crack advance either by simultaneous or alternating slip on these two systems. This damage process is the primary intrinsic mechanism promoting crack advance.

Simple models for striation formation [e.g., McClintock, 1967] predict that an upper-bound estimate for the increment of crack advance per cycle should be proportional to the cyclic crack tip opening displacement (Δ CTOD):

$$\frac{\mathrm{d}a}{\mathrm{d}N} \propto \Delta \mathrm{CTOD} \approx \beta \frac{\Delta K^2}{2\sigma_0 E'},\tag{3}$$

where σ_0 and E' are respectively the appropriate flow stress and Young's modulus, and β is a proportionality constant, of order 0.1-0.5, reflecting the efficiency of crack-tip blunting and reversibility of slip. This approach provides a first-order description of crack-growth rate behavior in the mid-range of growth rates (regime B in Figure 4), as shown for example by recent results on a bulk amorphous Zr-Ti-Cu-Ni-Be metal where crack advance occurs by a

striation mechanism (Figure 5) (Gilbert et al., 1997), although it is an insufficient description at high growth rates and in the near-threshold regime.

At high growth rates as $K_{\text{max}} \rightarrow K_c$ (regime C in Figure 4), Equation 2 underestimates measured growth rates due to the occurrence of monotonic fracture mechanisms (static modes) which replace or are additional to striation growth. Such mechanisms include cleavage, intergranular cracking and microvoid coalescence and their presence results in growth-rate behavior that is markedly sensitive to microstructure and K_{max} (or R) (Ritchie and Knott, 1973). Conversely, at very low growth rates where $\Delta K \rightarrow \Delta K_{\text{TH}}$ (regime A in Figure 4), Equation 2 overestimates measured growth rates and behavior becomes markedly sensitive to K_{max} , microstructure and environment; the K_{max} dependence in this regime, however, results primarily from crack closure. At such near-threshold levels, the scale of local plasticity (i.e., the plastic-zone size, r_y) approaches microstructural size-scales, and measured growth rates become less than an interatomic spacing per cycle, indicating that crack advance is not occurring uniformly over the entire crack front [e.g., Ritchie, 1977]. Crack-growth mechanisms in this regime (typically where r_y is smaller than the grain size) generally are faceted [e.g., Yoder et al., 1979], often being referred to as 'microstructurally sensitive' or 'crystallographic' fatigue, and reflect more of a single shear mode of crack advance with associated mode II plus mode I displacements, particularly in coarse planar-slip materials.

3.2.2. Extrinsic mechanisms

Although the primary mechanism motivating fatigue-crack extension in ductile materials, i.e., crack-tip blunting and resharpening, is intrinsic and controlled principally by ΔK (or more precisely the local plastic strain range), extrinsic crack closure mechanisms act in the crack wake to oppose this. Such wedge shielding (Ritchie, 1988) results from local deformation, fracture and chemical processes which induce physical contact between the mating crack surfaces at positive loads during the fatigue cycle. Elber (1970) originally proposed that closure arises from the constraint of surrounding elastic material on the residual stretch in material elements previously plastically strained at the tip (*plasticity-induced closure*). Since the crack cannot propagate while it remains closed, the net effect is to reduce the nominal (applied) ΔK value to some lower effective (local) value ΔK_{eff} actually experienced at the crack tip:

$$\Delta K_{\rm eff} = K_{\rm max} - K_{\rm cl}, \quad (K_{\rm min} \leqslant K_{\rm cl})$$
$$= K_{\rm max} - K_{\rm min}, \quad (K_{\rm cl} \leqslant K_{\rm min}), \tag{4}$$

where K_{cl} is the stress intensity to close the crack. There are, however, several mechanisms of closure which assume greater importance at near-threshold levels, where CTODs are small and approach the dimensions of the 'wedge'. These processes rely on wedging mechanisms inside the crack from corrosion debris, fracture surface asperities, or, in the case of environmentally assisted fatigue, fluid inside the crack, as reviewed in (Suresh and Ritchie, 1984).

Crack closure arising from crack surface corrosion deposits (*oxide-induced closure*) is promoted in oxidizing environments at low load ratios. Notable examples are the crack surface oxides and calcareous deposits formed during corrosion fatigue in structural steels tested, respectively, in water and sea water, and the chromic oxides formed during creep fatigue in Ni-based superalloys. Simple quantitative modeling, based on the concept of a rigid wedge inside a linear elastic crack, suggests that such closure depends upon the thickness d of the oxide film and the location of its peak thickness from the crack tip 2z (Suresh and Ritchie, 1984).

$$K_{\rm cl} \approx \frac{E'd}{4\sqrt{\pi z}},$$
(5)

implying that deposits in the immediate vicinity of the crack tip will have a dominating influence in the development of closure by this mechanism. A more general source arises from the wedging action of fracture surface asperities, where CTODs are small and where significant crack-tip shear displacements occur. Such *roughness-induced closure* is promoted at near-threshold levels, particularly where crack advance is strongly crystallographic, as in coherent-particle-hardened (planar slip) systems such as underaged Al alloys and Ni-based superalloys or in duplex microstructures where the crack can be made to meander from frequent crack deflection. Notable examples of where crack deflection and the resulting closure has led to excellent crack growth properties are found with dual-phase steels, β -annealed Ti alloys and Al-Li alloys. The magnitude of roughness-induced mechanism depends upon the degree of fracture surface roughness and the extent of the mode II crack-tip displacements. From simple two-dimensional geometric modeling, the closure stress intensity at the point of first asperity contact is given as (Suresh and Ritchie, 1984):

$$K_{\rm cl} \approx \sqrt{\frac{2\gamma u}{1+2\gamma u}} K_{\rm max},$$
 (6)

where γ is a measure of surface roughness, that is, ratio of height to width of the asperities, and *u* is the ratio of mode II to mode I displacements.

Since crack closure mechanisms involve a wedging process which effectively raises the minimum stress intensity, the potency of the shielding depends upon the magnitude of the crack-tip opening displacement compared to the dimension of the wedge. Since CTODs are much larger at R, the closure effect is markedly diminished at high load ratios, i.e., it is controlled by the value of K_{max} , particularly at lower ΔK levels where closure results predominantly from oxide or asperity wedging. Thus, in the fatigue of ductile materials, it is the ΔK -component of the driving force which controls the intrinsic damage mechanisms ahead of the crack tip¹ whereas the K_{max} -component controls the extrinsic shielding mechanisms behind the tip.

4. Fatigue-crack propagation in brittle ceramic materials

4.1. GENERAL CONSIDERATIONS

The role of extrinsic mechanisms is far more important in the toughening of materials with little or no ductility. Indeed, intrinsic toughening mechanisms, to promote crack-tip plasticity, for example, have been largely unsuccessful in brittle solids such as glasses and ceramics. Instead, ceramic materials have been toughened using a variety of crack-tip shielding mechanisms (Figure 6) [e.g., Evans, 1990; Becher, 1991]. Their effect is to impede crack extension via

¹ The exception to this is behavior at high stress intensities as K_{max} approaches K_c or the limit-load instability (Regime C in Figure 4), where additional static fracture mechanisms (e.g., cleavage, microvoid coalescence) occur; these mechanisms are primarily K_{max} controlled (Ritchie and Knott, 1973).



Figure 6. Schematic illustration of the primary toughening mechanisms in ceramics and ceramic-matrix composites. Note that all mechanisms are extrinsic in nature and promote inelastic deformation which results in a nonlinear stress/strain relationship.

mechanical, microstructural and environmental factors that *locally* reduce the near-tip stress intensity, i.e., to promote *R*-curve toughening.

For example, the intrinsic toughness of zirconia is $\sim 2 \text{ MPa}\sqrt{\text{m}}$. However, by inducing an *in situ* phase transformation at the crack tip (transformation toughening) or by causing the *in situ* microcracking of particles (microcrack toughening), both processes causing a dilation around the crack tip which is constrained by surrounding elastic material, the measured K_c can be raised extrinsically to between 8 and 13 MPa $\sqrt{\text{m}}$ [e.g., Evans, 1990]. Similarly, by inducing crack branching and meandering, due to crack deflection at particles or interfaces, factors of 3 increases in toughness can be obtained. Composites can be also toughened by crack-tip shielding mechanisms; a potent form is through crack bridging where cracks are bridged by intact brittle fibers or whiskers, or are made to intersect a ductile phase that undergoes plastic deformation as the crack passes [e.g., Evans, 1990; Becher, 1991].

With such shielding mechanisms, monolithic and composite ceramics can now be processed with toughnesses up to an order of magnitude higher than was available 20 or so



Figure 7. Fractography of ceramic fatigue showing nominally identical fracture surfaces under, respectively, monotonic and cyclic loading in (a,b) alumina (Coors 99.5%) and (c,d) silicon nitride (hot pressed with 7 wt.% $Al_2O_3 + Y_2O_3$). Note, however, the more debris and surface damage on the fatigue surfaces (courtesy of C.J. Gilbert).

years ago. However, it is ironic that whereas glasses and untoughened ceramics are essentially immune to cyclic fatigue (Evans and Fuller, 1974),² the generation of a nonlinear stress-strain curve with toughened ceramics results in their susceptibility to premature fatigue failure under cyclic loading. The characteristics of cyclic fatigue in ceramics appear to be quite different to metal fatigue:

- Unlike in ductile materials and with the exception of phase-transforming ceramics such as PSZ (Steffen et al., 1991), fatigue cracks in ceramics do not appear to initiate naturally; crack initiation is invariably associated with some pre-existing defect.
- Again unlike metal fatigue where there is a characteristic fracture mode for cyclic loading, i.e., striation growth, which is quite distinct from that for monotonic loading, the morphology of fatigue fracture surfaces in ceramics is almost identical to that under monotonic loads (Figure 7), although more debris is often present on the fatigue fractures.
- Microstructure can have a significant effect on fatigue-crack growth rates in ceramics, as for example shown by the variation in da/dN with ∆K for a series of aluminas with increasing *R*-curve toughness, resulting from increasing the grain sizes varying from ~ 1 to 30 µm (Gilbert and Ritchie, 1997). However, by normalizing the data in terms of the fracture toughness, i.e., by characterizing the growth rates with K_{max}/K_c, the microstructural effects are essentially removed (Figure 8). This implies that unlike metals, the microstructural influences on fracture by fatigue are similar to those for overload fracture.

 $^{^2}$ Whereas mechanical fatigue effects are generally non-existent in amorphous glasses and untoughened ceramics, recent studies (Dill et al., 1997) have identified a small fatigue effect in borosilicate glass at near-threshold growth rates, which is attributed to environment processes such as the entrapment of reacting water molecules.



Figure 8. Fatigue-crack growth rates, da/dN, as a function of (a) the applied stress-intensity range, ΔK , and (b) the maximum stress intensity normalized by the fracture toughness, K_{max}/K_c , for a range of polycrystalline aluminas (Gilbert and Ritchie, 1997).

The sensitivity of growth rates to the stress intensity is extremely high; specifically, the exponent *m* in the simple Paris equation (Equation 2) can take values as high as ~ 15 to 50 and above (Ritchie and Dauskardt, 1991), as shown by the growth-rate curves for alumina, PSZ and pyrolytic carbon in Figure 1. The very high exponents, however, result from a particularly marked sensitivity of growth rates to K_{max}, rather than ΔK per se. This can be appreciated by expressing the growth-rate relationship in terms of both K_{max} and ΔK, viz (Liu and Chen, 1991; Dauskardt et al., 1992):

$$da/dN = C'(K_{\rm max})^n (\Delta K)^p, \tag{7}$$

where, compared to Equation 2, C' is a constant equal to $C(1 - R)^n$ and (n + p) = m. In a typical brittle ceramic, e.g., SiC-whisker reinforced Al₂O₃, the exponents *n* and *p* are ~ 10 and 5 (Dauskardt et al, 1992), respectively; this is to be compared with values of n = 0.5 and p = 3 for metal fatigue of a nickel-base superalloy (Van Stone, 1988).

4.2. MECHANISTIC ASPECTS

In light of the factors described above, it is clear that mechanistically fatigue-crack advance in ceramics is conceptually different from that in metals. Again we can involve the intrinsic vs. extrinsic concept to explain this distinction. Essentially, there are two possible classes of fatigue mechanisms (where failure is associated with a dominant crack):

- *intrinsic mechanisms* where, as in metals, crack advance results from damage processes in the crack-tip region, which are unique to cyclic loading.
- *extrinsic mechanisms*, where the crack-advance mechanism ahead of the crack tip is identical to that for monotonic loading, but the unloading cycle promotes accelerated crack growth by degrading the degree of crack-tip shielding behind the tip.

Whereas the cyclic processes in metal fatigue are predominantly intrinsic in nature, the cyclic fatigue processes in ceramics are extrinsic. The mechanism by which the crack advances is thus identical under cyclic loading as it would be in a single overload cycle; this clearly is consistent with the marked dependency of growth rates on K_{max} (rather than ΔK) and a similarity in fracture surface appearance under cyclic and monotonic loading. The cyclic loading conversely acts to diminish the shielding (i.e., the *R*-curve toughening) in the crack wake. This can take various forms depending upon the prevailing shielding mechanism; examples include the premature fatigue failure of metallic reinforcement phase in ductile-phase toughened materials (Venkateswara Rao et al., 1992a; 1994), a reduction in the effect of fiber bridging in fiber-reinforced ceramic-matrix composites (Rouby and Reynaud, 1993), and a decreased bridging capacity of a wake zone of interlocking grains in grain-bridging ceramics, such as coarse-grained Al₂O₃, grain-elongated Si₃N₄, and *in situ* toughened SiC (Dauskardt and Ritchie, 1991; Lathabai et al., 1991; Dauskardt, 1993; Kishimoto et al., 1995; Gilbert et al., 1996; Gilbert and Ritchie, 1998).

To illustrate this phenomenon, the latter mechanism of a fatigue-induced reduction in grain bridging in monolithic ceramics is considered (Fgure 9). As the decay in bridging is associated with the relative motion of the grains as the crack proceeds intergranularly, this can be modeled in terms of cycle-dependent sliding wear degradation of frictional grain bridges (Dauskardt, 1993).

Specifically, where thermal expansion anisotropy results after processing in certain grains experiencing a state of residual compression, cracks will extend preferentially in the tensile regions surrounding the grain, leaving it intact and spanning the crack. The resulting closing traction on the crack surfaces, the grain bridging stress, p, can be expressed as a function of the distance behind the crack tip X, and the crack-opening displacement 2u, in terms of the length of the bridging zone L, and an exponent k, as (Evans, 1990; Lawn, 1993; Hay and White, 1993; Mai and Lawn, 1987; Foote et al., 1986):

$$p(X) = P_{\max}(1 - X/L)^{k},$$

$$p(u) = P_{\max}(1 - X(u)/L)^{k}.$$
(8)



Figure 9. Crack-tip shielding from (a) bridging by interlocking grains, showing (b) the variation in bridging tractions p(u) due to debonding and frictional pullout, as a function of the crack opening displacement (2u), and (c) the decay in such bridging under cyclic loading.

The function p(x) describes the bridging stress distribution as a function of distance behind the crack tip X starting from a maximum value of P_{max} at the crack tip (X = 0), and falling to zero at the end of the bridging zone (X = L), where $2u = 2u_f$, the critical crack-opening for bridge rupture. The maximum bridging stress is related to the residual stresses and is equivalent to the product of the residual clamping stress acting on a grain σ_N , due to thermal expansion anisotropies and the coefficient of friction μ , i.e., $P_{\text{max}} = \mu \sigma_N$. The shape of the decrease is determined by the exponent k; a uniformly distributed stress over the bridging zone would imply k = 0 whereas k = 1 corresponds to frictional pullout where the cross-sectional geometry of the bridging grains does not change during crack opening (Foote et al., 1986).

The bridging stress p(u) rises rapidly with u during debonding of the grain, followed by a gradual decrease where frictional pullout of the grain occurs (Figure 9); both processes contribute to the toughness, although in general most energy is dissipated during frictional pullout. Under cyclic loads, the repetitive opening and closing of the crack results in a decrease in the toughening capacity of the bridging zone by reducing the grain bridging stress, i.e., accumulated damage at the grain/matrix interface ahead of the tip and reduced frictional sliding resistance of partially debonded grain/matrix interfaces from frictional wear causes premature grain debonding and a reduced frictional pullout stress.

A quantitative verification of this extrinsic mechanism for fatigue-crack growth in ceramics can be achieved by measuring the crack-opening profiles of cracks grown at high velocity near the K_c instability (to simulate behavior on the *R*-curve) and at low velocity near the fatigue threshold (to simulate the cyclically-loaded crack) (Gilbert and Ritchie, 1998). The net crackopening profile, u(X), for a linear-elastic crack under an applied far-field stress intensity K_A , with a bridging traction distribution p(X), of length *L* acting across the crack faces can be



Figure 10. Measured crack-opening profiles in an ABC-SiC ceramic for (a) Case I where the crack was grown near instability (at $\sim 2 \times 10^{-8}$ m/cycle at $\sim 92\%$ K_c), and Case II where the crack was grown near threshold (at $\sim 1 \times 10^{-10}$ m/cycle at $\sim 75\%$ K_c). In (b), the best-fit bridging traction distributions are plotted for each case (Gilbert and Ritchie, 1998).

expressed, in terms of the elastic modulus E' (= E in plane stress or $E/(1 - v^2)$ in plane strain; v is Poisson's ratio), as (Barenblatt, 1962):

$$u(X) = \frac{K_A}{E'} \left(\frac{8X}{\pi}\right)^{1/2} + \frac{2}{\pi E'} \int_0^L p(X') \ln \left| \frac{\sqrt{X'} + \sqrt{X}}{\sqrt{X'} - \sqrt{X}} \right| \, \mathrm{d}X',\tag{9}$$

where X' is the integrated variable where the stress p acts. The first term in Equation 9 reflects the *traction free* crack under tensile loading in small-scale yielding (Irwin, 1958).

Scanning electron microscopy measurements on an *in situ* toughened silicon carbide (ABC-SiC) of the opening profile for a crack approximating *R*-curve behavior (grown at $K_A = 6.22 \text{ MPa}/\text{m}$ (~ 92% K_c)) are plotted in Figure 10a as function of distance X behind the crack tip (Gilbert and Ritchie, 1998). Also shown are the best-fit profile determined from



Figure 11. Crack opening profile measurements in ABC-SiC ceramic from Figure 10(a) are used to simulate (a) the *R*-curve under monotonic loading, and (b) the fatigue-crack growth rates under cyclic loading (Gilbert and Ritchie, 1998).

Equation 9 (dashed line) and the calculated opening profile for a traction-free crack (solid line). Using fitting procedures on these data, the best-fit p(X) and u(X) functions can be estimated (Figure 10b) and used to calculate the p(u) function (Equation 8), from which the *R*-curve can be predicted (Gilbert and Ritchie, 1998). The bridging contribution for a crack that has propagated an amount Δa , $K_B(\Delta a)$, can be estimated from p(X) via (Lawn, 1993):

$$K_B(\Delta a) = \left(\frac{2}{\pi}\right)^{1/2} \int_0^{\Delta a} \frac{p(X)}{\sqrt{X}} \,\mathrm{d}X.$$
(10)

By varying Δa from $\Delta a = 0$ to $\Delta a = L$, and knowing long-crack initiation toughness $(K_0 \sim 5.5 \text{ MPa}_{\sqrt{m}} \text{ for ABC-SiC}$ (Gilbert et al., 1996), the resistance curve $K_R(\Delta a)$ can be determined from:

$$K_R(\Delta a) = K_0 + K_B(\Delta a). \tag{11}$$

Predictions for ABC-SiC are shown in Figure 11(a) and can be seen to be in reasonable agreement to that measured experimentally on compact-tension specimens (Gilbert and Ritchie, 1998). The corresponding opening profile (at $K_A = 6.22 \text{ MPa}/\text{m}$) for a crack grown after extensive fatigue cycling at a near-threshold growth rate of $\sim 1 \times 10^{-10}$ m/cycle is compared in Figure 10a with the profiles for the traction-free (solid line) and *R*-curve cracks. It is apparent that the near-threshold crack is significantly more open than the *R*-curve crack, particularly in the near crack-tip region where most of the steady-state toughness is developed (i.e., within $\sim 600 \,\mu\text{m}$ of the tip); indeed, the profile is essentially identical to that for the traction-free crack. This is consistent with a reduction in the magnitude of bridging tractions for a bridging zone that has experienced $\sim 10^8$ loading cycles. Best-fit p(X) functions from the fatigue and *R*-curve cracks are compared in Figure 10b, and show how continued cyclic loading acts to degrade the bridging tractions (Gilbert and Ritchie, 1998).

By estimating the magnitude of bridging tractions at several applied K levels using this crack profile method, the cyclic crack growth-rate data can be simulated based on the notion that the crack-advance mechanism does not change and that the influence of the cyclic loads is solely to progressively degrade crack bridging., i.e., the *local* crack-tip stress intensity remains at K_0 . The predicted variation in da/dN with ΔK for the ABC-SiC can be seen in Figure 11b to be in reasonably close agreement with experimentally measured data at R = 0.1 (Gilbert and Ritchie, 1998). The closeness of this fit in addition to the direct quantitative evidence of a cycle-induced degradation in shielding imply that the essential physics of the cyclic-crack growth process involves the cycle-dependent degradation of the active bridging zone.

5. Fatigue-crack propagation in intermetallic materials

5.1. GENERAL CONSIDERATIONS

Due to their complex, and invariably ordered, crystal structures, intermetallics like ceramics generally display only very limited mobile dislocation activity at low homologous temperatures (below their brittle-to-ductile transition temperature, DBTT), and are thus often highly restricted in ductility and toughness. To toughen intermetallics, both intrinsic and extrinsic toughening approaches have been attempted, and to varying degrees have been successful [e.g., Lui et al., 1989; Pope et al., 1995; Soboyejo et al., 1995a]. For example, the intermetallic compound Nb₃Al can be toughened through the addition of a ductile phase such as Nb, with the intermetallic matrix cracking preferentially to the metallic reinforcement (Murugesh et al., 1993; Cao et al., 1994; Bloyer et al., 1996). In fact, using a high-aspect ratio reinforcement, e.g., a Nb layer in an arrester laminate, the crack-initiation toughness may be enhanced *intrinsically* by the necessity of the crack in the intermetallic matrix renucleating across the metal phase. However, in addition, as the crack extends leaving uncracked Nb ligaments in its wake, the crack-growth (or *R*-curve) toughness is also enhanced, in this case *extrinsically* from the crack bridging created by the zone of intact ductile ligaments behind the crack tip.

In many respects, the mechanical behavior of intermetallics can be considered to be intermediate between metals and ceramics, ranging from compounds with some ductility, such as α_2 -Ti₃Al, that display 'metal-like' characteristics, to the very brittle Nb₃Al and MoSi₂ that are 'ceramic-like' (below the DBTT). Specifically, with respect to fatigue-crack propagation, intrinsic damage mechanisms associated with crack advance do appear to operate in the more ductile intermetallics as in metals (although they have not been studied in detail); in contrast, the mechanism by which the crack extends in the cyclic fatigue of brittle intermetallics, such as Ni₃Al and MoSi₂, is identical to that occurring under monotonic loads, as in ceramics [e.g., Venkateswara Rao et al., 1992b; Hong et al., 1998]. Moreover, whereas the intrinsic

toughening mechanisms, such as crack renucleation, do not degrade under cyclic loading, the extrinsic toughening mechanisms, such as crack bridging, can suffer severe degradation. A notable example is ductile-phase reinforced intermetallic composites, which due to extensive crack bridging by the uncracked ductile phase, can display significantly higher toughness (i.e., by a factor of 3 or greater) than the constituent matrix [e.g., Venkateswara Rao et al., 1992a; Murugesh et al., 1993]. However, the improvement in crack-growth resistance is far less obvious in fatigue simply because the ductile phase fails prematurely; indeed, the fatigue-crack growth properties are rarely much better than that of the unreinforced matrix (Venkateswara Rao et al., 1992a; 1994). Thus, similar to ceramic materials, cyclic loading can act to reduce the potency of extrinsic toughening mechanisms in impeding crack advance.

Compared to ceramics and especially metals, the fatigue of intermetallics has not been studied extensively [e.g., Stoloff, 1996]. For example, few microstructures or compositions in intermetallic systems have ever been optimized for fatigue strength. However, in light of the cyclic loading induced decay in extrinsic toughening during fatigue-crack growth, the only approaches that are available are to develop intrinsic toughening that will not degrade, which to date has not been too successful, or to devise extrinsic mechanisms that are more resilient to alternating loads. Below we briefly explore the latter approach, again in the context of fatigue-crack growth being a compromise between intrinsic and extrinsic processes, by examining the behavior of three intermetallic systems based on γ -TiAl, MoSi₂ and Nb₃Al.

A good example of this behavior is shown by γ -TiAl alloys reinforced with small fractions of ductile Nb or TiNb, where toughening derives from tractions induced by unbroken ductile ligaments bridging the crack wake (Figure 12) (Venkateswara Rao et al., 1992a; 1994). For small-scale bridging, the toughness increases with crack extension up to a maximum steadystate level K_c associated with the development of a steady-state bridging zone, and can be estimated from the flow stress σ_0 , volume fraction f, and size t of the reinforcement (Ashby et al., 1989):

$$K_c = \sqrt{K_o^2 + E' f t \sigma_o \chi},\tag{12}$$

where K_o is the critical crack-tip stress intensity at crack initiation, E' is the appropriate elastic modulus of the composite, and χ is a dimensionless function representing the work of rupture. For γ -TiAl reinforced with Nb or Nb-alloys, χ varies between 0.9 and 1.5; much larger values of $\chi(< 4)$ can be obtained using strain-hardening reinforcements that debond readily from the matrix (Dève et al., 1990). For nominal values of $\chi = 1.2$, $\sigma_0 = 400$ MPa and $t = 100 \,\mu\text{m}$, the addition of a mere 20 vol.% of ductile particles yields $K_c \sim 44 \,\text{MPa}$ \sqrt{m} , over five times the toughness of unreinforced TiAl (Venkateswara Rao et al., 1994). Under cyclic loads, however, such bridging from unbroken ductile ligaments is compromised by the susceptibility of the ductile phase to premature fatigue failure. Accordingly, despite the factor of 3–5 increase in toughness, the fatigue-crack growth properties of the composite are comparable (and in specific orientations worse) than that of the unreinforced matrix (Venkateswara Rao et al., 1992a; 1994). However, Nb reinforcements, which undergo extensive interface debonding (due to σ -phase in the reaction layer), can delay this failure process. Nb thus provides improved fatigue resistance over reinforcements such as TiNb which are more strongly bonded to the matrix (due to an α_2 -Ti₃Al reaction layer) (Figure 12) (Venkateswara Rao et al., 1994).



Figure 12. (a) Fracture toughness and (b) fatigue-crack growth behavior in a 20 vol.% TiNb-reinforced γ -TiAl intermetallic composite at 25°C at R = 0.1 in the edge (C-R) and face (C-L) orientations, where the results are compared to data for pure γ -TiAl, β -TiNb and a Nb-particulate reinforced γ -TiAl composite. (c) Extensive crack bridging by the uncracked ductile phase under monotonic loading is severely degraded under cyclic loading due to (d) premature fatigue failure of the ductile ligaments (Venkateswara Rao et al., 1992a; 1994).

Thus, to achieve good fatigue-crack propagation resistance in intermetallics, it is necessary to develop toughening mechanisms that are more resilient to cyclic loading. For example, for ductile-phase toughened systems, several strategies are available and involve the use of:

- *high aspect-ratio reinforcements*; this merely increases the probability of interception by the crack. For example, MoSi₂ reinforced with spherical Nb particles displays insignificant toughening and no improvement in fatigue resistance as the crack merely circumvents the ductile phase (Venkateswara Rao et al., 1992b); in contrast, with Nb wire-mesh (Badrinarayanan et al., 1996) or laminate (Soboyejo et al., 1995b) reinforcements, a 3-fold increases in toughness (due to extensive crack bridging) and good fatigue-crack growth resistance (due to crack closure) can be obtained (Figure 13).
- reinforcements with weakened interfaces with the matrix; the resulting delamination at the interface acts to delay the inevitable failure of the ductile ligament, as shown by the improved fatigue properties of Nb/ γ -TiAl compared to TiNb/ γ -TiAl (Figure 12) (Venkateswara Rao et al., 1994).
- reinforcements with good intrinsic fatigue resistance; whereas β -TiNb is a useful reinforcement phase for toughening γ -TiAl because of its high strength ($\sigma_0 \sim 430$ MPa),



Figure 13. (a) Ductile-phase reinforcement of molybendum disilicide with niobium leads to (b) significant *R*-curve toughening with a high-aspect ratio (wire-mesh) Nb phase, due to crack bridging by the intact Nb (Badrinarayanan et al., 1996); only minimal toughening is achieved with corresponding particulate reinforcements (Bloyer et al., 1996). (c) Although such bridging in the Nb wire-mesh reinforced composite is degraded under cyclic loading, the meandering crack path (along the brittle Nb/matrix reaction layer) promotes alternative shielding from roughness-induced crack closure and good fatigue-crack growth resistance (Badrinarayanan et al., 1996).



Figure 14. (a) Crack-tip shielding by uncracked (shear) ligament bridging in ($\gamma + \alpha_2$) titanium aluminade (e.g., Ti-47wt.%Al) alloys promotes excellent (b) *R*-curve toughening and (c) fatigue-crack growth resistance (compared to single-phase γ), particularly in the coarser lamellar microstructures. Such bridging does degrade under cyclic loading; however, the uncracked ligaments do not necessarily fail preferentially (as in ductile-phase bridging) as they are the same material as the matrix (Campbell et al., 1998).



(c).

Figure 15. (a) Fatigue-crack propagation rate behavior in Nb-layer reinforced niobium aluminide laminates, where good crack-growth resistance is achieved in the coarser-scale arrester laminates by (b) intrinsic toughening, i.e., crack renucleation across the Nb layer (note the opposing directions of the river markings on the cleavage fracture of the reaction layer), and (c) extrinsic toughening, i.e., crack bridging by unbroken Nb layers. Note that unlike the extrinsic bridging mechanism, the intrinsic mechanism does not degrade under cyclic loading (Bloyer et al., 1996; 1997). Horizontal arrows represent direction of crack growth in (b) and (c).

its fatigue properties are poor due to its near zero strain hardening which induces rapid shear localization. A better approach is to use monolithic γ -TiAl alloys with lamellar microstructures, where (*R*-curve) toughening results from uncracked (shear) ligament bridging (Chan, 1995); here, since the bridge material is the same as the matrix, there is a lower tendency for the bridges to fail preferentially in fatigue (Figure 14) (Campbell et al., 1998).

- reinforcements that promote other extrinsic toughening mechanisms; since bridging mechanisms invariably fail under fatigue loading, good fatigue-crack growth resistance can be achieved if alternative shielding mechanisms develop. For example, with Nb-wire mesh reinforced MoSi₂, the Nb bridging ligaments which provide for *R*-curve toughening do fail prematurely in fatigue; however, as the crack follows the weakened interfacial reaction layer of the Nb phase, highly tortuous crack paths result which give rise to high levels of roughness-induced crack closure (Figure 13) (Badrinarayanan et al., 1996).
- *reinforcements that promote intrinsic toughening*; whereas extrinsic shielding mechanisms tend to degrade in fatigue, intrinsic toughening mechanisms do not. Accordingly, the use of laminate reinforcements in the arrester orientation, which require the crack to renucleate across the ductile layer, both increase the crack-*initiation* toughness and remain equally potent under cyclic loading. Such behavior is shown by coarser-scale Nb/Nb₃Al laminated composites (Figure 15) (Bloyer et al., 1996; 1997).

6. Differences and similarities between the fatigue of ductile and brittle materials

Compared to the extensive database and understanding of fatigue failure in metals (e.g., Suresh, 1991; Ellyin, 1997; Davidson and Lankford, 1992; Ritchie, 1979), ceramics and intermetallic systems still require extensive research with respect to fatigue behavior. In such materials, it is now clear that cyclic loading induces a progressive degradation in the toughening (or shielding) mechanisms behind the crack tip that locally elevates the near-tip driving force. It is this cyclic suppression of shielding that is considered to be the principal source for the susceptibility of brittle materials involves primarily *intrinsic* damage processes occurring *ahead* of the crack tip, i.e., involving crack advance by progressive blunting and resharpening of the crack tip, clearly a mechanism distinct from fracture under monotonic loads. Additionally, shielding, in the form of crack closure (wedging) mechanisms, can act in the crack wake.

Since the physical mechanisms of crack advance and crack-tip shielding are quite different in metals and ceramics, the dependencies on the alternating and mean loads, specifically ΔK and K_{max} , the alternating and maximum stress intensities, respectively, are also quite different. A schematic illustration highlighting these differences is shown in Figure 16. Clearly, in metals the dominant dependence of ΔK is a consequence of the intrinsic crack-advance mechanism; the smaller K_{max} dependence results primarily from its effect of the crack-opening displacement, which in turn controls the degree of crack wedging due to crack closure in the wake. Thus, for ductile metals, $n \ll p$ in Equation 7. In ceramics, conversely, growth rates are principally a function of K_{max} , since the crack-advance mechanism is identical to that under static loading; the much weaker ΔK dependence here arises from the cyclicinduced degradation in shielding in the wake. Thus, for ceramics, $p \ll n$. In intermetallics,



Figure 16. Schematic illustrations of the intrinsic and extrinsic mechanisms involved in cyclic fatigue-crack growth in (a) metals and (b) ceramics, showing the relative dependencies of growth rates, da/dN, on the alternating, ΔK , and maximum, K_{max} , stress intensities.

fatigue properties are intermediate between these two extremes, such that generally $p \sim n$. However, in brittle intermetallics such as MoSi₂, where there is no intrinsic cycle-dependent crack advance mechanism, p > n, whereas in more ductile materials, such as the $(\gamma + \alpha_2)$ TiAl alloys, intrinsic fatigue damage mechanisms, similar to those in metals, clearly exist, and p < n.

Such differing dependencies on ΔK and K_{max} also have marked influence on how the load ratio affects growth rates. In metals where the ΔK term is dominant, characterizing growth rates in terms of ΔK invariably normalizes the load-ratio dependence (at least in the intermediate range of growth rates from $\sim 10^{-9}$ to 10^{-6} m/cycle) (e.g., Ritchie, 1979). In ceramics, conversely, this normalization is achieved by characterizing growth rates as a function of K_{max} as this is dominant term (Gilbert et al., 1996), whereas in intermetallics, the load ratio dependence cannot be normalized by either ΔK of K_{max} (Badrinarayanan et al., 1996; Campbell et al., 1998; Bloyer et al., 1997).

Although both intermetallics and ceramics are susceptible to fatigue-crack growth from degradation of shielding under cyclic loads, there are significant differences. At ambient temperatures, there is no intrinsic fatigue damage mechanism in ceramics – the crack tip advances by an identical mechanism under monotonic and cyclic loads; at a given stress intensity, fatigue cracks simply grow faster due to the cyclic suppression in shielding behind the crack tip. Accordingly, the value of maximum stress intensity at the fatigue threshold, $K_{\text{max,TH}}$, is comparable with the crack-initiation toughness, K_o , at the start of the *R*-curve ($K_{\text{max,TH}} \sim K_o$). In many intermetallics, conversely, fatigue-crack growth is seen at stress-intensity levels below the crack-initiation toughness, specifically $K_{\text{max,TH}} \sim 0.25 - 0.4 K_o$, implying there are additional *intrinsic* microstructural damage mechanisms associated with fatigue failure; however,

this effect is more restricted than in metals, where generally $K_{\text{max,TH}} \leq 0.1 K_o$, due to the limited crack-tip plasticity and consequently lower toughness of intermetallic alloys.

7. Fatigue design and life prediction

The marked sensitivity of fatigue-crack growth rates to the applied stress intensity in intermetallics and ceramics, both at elevated and especially ambient temperatures, presents unique challenges to damage-tolerant design and life-prediction methods for structural components fabricated from these materials. For safety-critical applications involving most metallic structures, such procedures generally rely on the integration of data relating crack-growth rates (da/dN or da/dt) to the applied stress intensity (ΔK or K_{max}) in order to estimate the time or number of cycles N_f to grow the largest undetectable initial flaw a_i to critical size a_c , viz:

$$N_f = \frac{2}{(m-2)C(Q\Delta\sigma)^m \pi^{m/2}} [a_o^{(2-m)/2} - a_c^{(2-m)/2}],$$
(13)

where $\Delta \sigma$ is the applied stress range, Q is the geometric factor; C and $m \neq 2$) are the scaling constant and exponent in the crack-growth relationship in Equation 2. For advanced materials such as intermetallics and ceramics, this approach may be difficult to implement in practice due to the large values of the exponent m; since the projected life is proportional to the reciprocal of the applied stress raised to the power of m, a factor of two change in applied stress can lead to projections of the life of a ceramic component (where m can be as high as 20 or more) to vary by more than six orders of magnitude. Essentially, because of the high exponents, the life spent in crack propagation in advanced materials will be extremely limited or infinitely large, depending upon whether the initial stress intensity is above or below the fatigue threshold.

Accordingly, a more appropriate approach may be to design on the basis of a threshold below which fatigue failure cannot occur; this may involve a conventional fatigue-crack initiation threshold or fatigue limit determined from stress-life (S/N) data or, more conservatively, the ΔK_{TH} or $K_{\text{max,TH}}$ thresholds for no crack growth. However, even these approaches may not be conservative due to uncertainties in the definition of such thresholds. This is particularly important in many advanced materials as their growth-rate curves are not sigmoidal and correspondingly show little evidence of a threshold (Figures 1, 8, 11–15). Moreover, as with metallic materials [e.g., Ritchie and Lankford, 1986; Miller and de los Rios, 1992], there is now increasing evidence for small-crack effects in both ceramics (Steffen et al., 1991; Dauskardt et al., 1992) and intermetallics (Campbell et al., 1997), where cracks of a size comparable with the scale of microstructure, the extent of local inelasticity ahead of the crack tip, or with the extent of shielding in the wake of the tip, can propagate at applied stress intensities below the 'long-crack' ΔK_{TH} or $K_{\text{max,TH}}$ thresholds. Clearly for flaw-sensitive materials such as ceramics and intermetallics to be safely used in fatigue-critical applications, the critical levels of damage necessary for the *onset* of fatigue failure must be defined; whether this involves an S/N fatigue limit, long- or small-crack thresholds, or more refined statistical analyses is as yet still unclear.

8. Summary and conclusions

Major progress has been made over the last ten to fifteen years in significantly improving the fracture resistance of low-ductility materials such as ceramics and intermetallics using

	$\mathrm{d}a/\mathrm{d}N \propto (K_{\mathrm{max}})^n (\Delta K)^p$	relationship of $K_{\max,TH}$ to K_o^*
Metals Intermetallics Ceramics	$p \gg n$ $p \sim n$ $p \ll n$	$K_{ m max,TH} \leqslant 0.1 K_o$ $K_{ m max,TH} \sim 0.25 - 0.4 K_o$ $K_{ m max,TH} \sim K_o$

Table 1. Differences in the Fatigue-Crack Growth Properties of Ductile and Brittle Materials

 $*K_{\text{max,TH}}$ and K_o are, respectively, the fatigue threshold and the crack-initiation toughness on the *R*-curve.

the extrinsic shielding approach to toughening. This has included the design of microstructures with develop zones of inelasticity, microcracking or most predominantly bridging (by grains, particulate, fibers or layers) that surround the crack. However, it is now evident that cyclic fatigue loading can severely degrade such toughening; in fact, this provides the critical mechanism promoting fatigue-crack growth, even in materials such as ceramics that display no intrinsic mechanism of cyclic crack advance.

Although the mechanisms of cyclic fatigue in brittle materials are conceptually different from the well known mechanisms of metal fatigue, the central thesis of this paper is that by considering the relative importance of the prevailing intrinsic damage and extrinsic shielding mechanisms, a commonality of behavior can be found for the cyclic fatigue of both ductile and brittle materials. However, as noted above, the respective contributions of each mechanism does result in marked differences in the ΔK and K_{max} dependence of growth rates, the load ratio effect and the relationship of the fatigue threshold to the toughness properties; these are summarized in Table 1.

Finally, the marked sensitivity of growth rates to the applied stress intensity in ceramics and intermetallics implies that projected lifetimes will be a very strong function of stress and crack size; this makes design and life prediction using damage-tolerant methodologies more complex. Accordingly, design approaches based on the definition of a critical level of damage for the *onset* of fatigue cracking must be contemplated, involving either an S-N fatigue limit or crack-propagation threshold, although even these approaches may not be conservative due to the sub-threshold extension of small cracks. Clearly, this is an area that mandates increased attention in the future if these materials are to find widespread structural use.

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