

# Solution to the problem of the poor cyclic fatigue resistance of bulk metallic glasses

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The recent development of metallic glass-matrix composites represents a particular milestone in engineering materials for structural applications owing to their remarkable combination of strength and toughness. However, metallic glasses are highly susceptible to cyclic fatigue damage, and previous attempts to solve this problem have been largely disappointing. Here, we propose and demonstrate a microstructural design strategy to overcome this limitation by matching the microstructural length scales (of the second phase) to mechanical crack-length scales. Specifically, semisolid processing is used to optimize the volume fraction, morphology, and size of second-phase dendrites to confine any initial deformation (shear banding) to the glassy regions separating dendrite arms having length scales of  $\approx 2 \mu\text{m}$ , i.e., to less than the critical crack size for failure. Confinement of the damage to such interdendritic regions results in enhancement of fatigue lifetimes and increases the fatigue limit by an order of magnitude, making these "designed" composites as resistant to fatigue damage as high-strength steels and aluminum alloys. These design strategies can be universally applied to any other metallic glass systems.

composites | damage confinement | endurance limit | semisolid processing

Monolithic bulk metallic glasses (BMGs) have emerged over the past 15 years as a class of materials with unique and unusual properties that make them potential candidates for many structural applications (1). These properties include their near theoretical strengths combined with high formability, low damping, large elastic strain limits, and the ability to be thermoplastically formed into precision net shape parts in complex geometries (2, 3), all of which are generally distinct from, or superior to, corresponding crystalline metals and alloys. However, monolithic BMGs can also display less desirable properties that have severely restricted their structural use. In particular, properties limited by the extension of cracks, such as ductility, toughness, and fatigue, can be compromised in BMGs by inhomogeneous plastic deformation at ambient temperatures where plastic flow is confined in highly localized shear bands (4, 5). Such severe strain localization with the propagation of the shear bands is especially problematic under tensile stress states where catastrophic failure can ensue along a single shear plane with essentially zero macroscopic ductility (6, 7). Consequently, resulting plane-strain  $K_{Ic}$  fracture toughnesses in monolithic BMGs are often low ( $\approx 15\text{--}20 \text{ MPa}\sqrt{\text{m}}$ ), as compared with most crystalline metallic materials, although they are an order of magnitude larger than those for (ceramic) oxide glasses (8, 9). If such strain localization is suppressed such that plastic flow is allowed to be extensive, for example, by blunting the crack tip, then damage would be distributed over larger dimensions with toughness values increasing to  $\approx 50 \text{ MPa}\sqrt{\text{m}}$  or more (8, 10). Whereas some metallic glasses appear to be intrinsically brittle in their as-cast state (11), others become severely embrittled on annealing due to structural relaxation and associated loss of free volume, elastic stiffening, or increasing yield strength, all leading to a reduction in the fracture toughness to values as low as those of ceramic glasses (11–15).

In addition to having questionable tensile ductility and toughness, monolithic BMGs are particularly susceptible to damage caused by cyclic loading. Although the macroscale crack propagation rate behavior is generally comparable to that for crystalline metals and alloys (10, 16), the fatigue resistance in terms of the  $10^7$ -cycle endurance strength (or fatigue limit) tends to be particularly low for metallic glasses in both bulk and ribbon form (17–22). Measurements on Zr-based glasses, for example, reveal a fatigue limit\* in four-point bending of  $\approx 1/10$  of the (ultimate) tensile strength or lower (20–22), in contrast with most crystalline metallic materials where fatigue limits are typically between  $1/2$  and  $1/3$  of their tensile strengths. Given the high strength ( $\approx 1 \text{ GPa}$  or more) of many metallic glasses and their known resistance to the initiation of plastic flow under monotonic loading, these observations of very low fatigue limits are both surprising and disappointing.

We reason that the low fatigue limits result simply from the lack of microstructure in monolithic BMGs; the incorporation of a second phase in monolithic BMGs would therefore provide a potential solution. Indeed, with the recent development of in situ BMG-matrix composites, the problems of poor ductility and toughness in BMGs have been mitigated by the presence of such a second phase that provides a means to arrest the propagation of shear bands (23–26). However, to date, attempts to similarly enhance the corresponding fatigue resistance have been largely unsuccessful (27–29). In fact, one study (29) found that the fatigue life was actually reduced, compared with the monolithic glass, after incorporation of a second dendritic phase. We believe that the disappointing results obtained so far are because inadequate attention has been paid to the dimensions of the incorporated microstructure. Accordingly, we demonstrate here that, by introducing a second phase in the form of crystalline dendrites and by creating an effective interaction between the length scales of the shear bands and that of the dendrites, the fatigue limit can be raised significantly, by as much as an order of magnitude, to approach values comparable to that of high-strength crystalline metallic materials.

## Results

Here, we examine a  $\text{Zr}_{39.6}\text{Ti}_{33.9}\text{Nb}_{7.6}\text{Cu}_{6.4}\text{Be}_{12.5}$  BMG-matrix composite that was developed for high toughness (26); this alloy, termed DH3, comprises crystalline ( $\beta$ -phase) dendrites within an amorphous matrix. In earlier versions of such composite alloys, cooling rate variations within the ingots caused large differences in the overall dendrite length scale, with interdendrite spacings varying by 2 orders of magnitude (from  $\approx 1$  to 100

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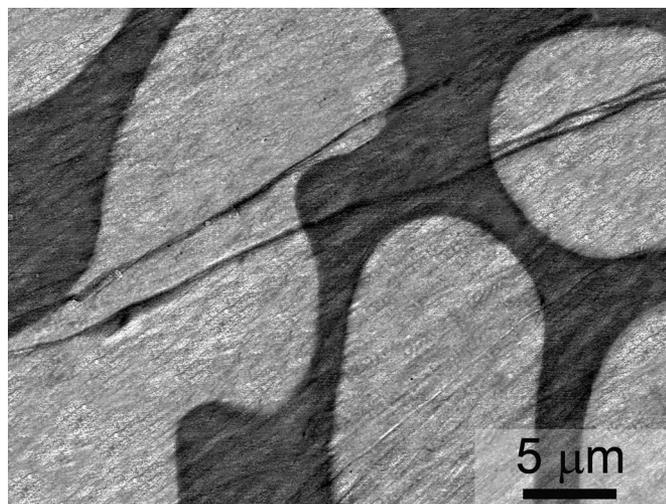
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\*The fatigue limit is expressed here in the usual way in terms of the applied stress amplitude,  $\sigma_a$ , which is defined as  $1/2(\sigma_{\text{max}} - \sigma_{\text{min}})$ , where  $\sigma_{\text{max}}$  and  $\sigma_{\text{min}}$  are, respectively, the maximum and minimum applied stresses in the loading cycle. The alternating stress is one-half of the stress range,  $\Delta\sigma$ .







**Fig. 5.** Dormant shear bands: scanning electron microscopy back-scattered electron image of the cross-section of a beam tested at the fatigue limit after  $2 \times 10^7$  cycles. Shear bands are observed near the tensile surface. Damage evolution occurs very early after only a few cycles. Some studies (16, 21) have suggested that the low fatigue limit reported for bulk metallic glasses may be associated with the presence of preexisting, micrometer-sized surface shear bands. In the current alloy, such shear bands are constrained by the crystalline second-phase dendrites to a length where they remain essentially dormant at the given stress amplitude,  $\sigma_a/\sigma_{UTS} \approx 0.3$ . The high fatigue limit of this material lies in its ability to provide microstructural barriers necessary to avoid propagation of the damage to critical size.

in both the dendrite and the glassy phases as seen in Fig. 3*D*. The cavitation during a stress cycle therefore must occur within an individual striation.

How does the above discussion relate to fatigue limits? For crystalline metals, fatigue lifetimes are largely dominated by the loading cycles required to initiate damage as opposed to propagating a “fatal” crack. The term initiation, however, is often a misnomer, because the rate-limiting process is generally not crack initiation but rather early propagation of small (often preexisting) flaws through a dominant microstructural barrier, e.g., a grain boundary or hard second-phase particle (38, 39). The lower fatigue limits of amorphous alloys can be attributed to the lack of a microstructure that provides local arrest points for newly initiated or preexisting cracks (16, 20, 21). Small cracks are observed to initiate after only a few stress cycles in BMGs (21). In contrast to crystalline alloys, fatigue lifetimes should therefore be governed by early crack propagation (rather than initiation), specifically by the number of cycles to extend a small flaw to some critical size (Fig. 5). In the present case of the BMG-matrix composite, the critical flaw size must be greater than some feature of the dendritic microstructure (i.e., the interarm spacing) to prevent unstable crack propagation.

To prevent a shear band from opening and causing failure between dendrite arms, the shear-band length must fall below a critical size that is determined by the applied stress and fracture toughness of the BMG. For high-cycle fatigue resistance, the dendrites must also limit microcrack growth (during  $10^7$  cycles) in the fatigue limit to a similar length. We illustrate this argument with a simple fracture-mechanics calculation. Considering the interdendritic shear bands (Fig. 3*A*) as small cracks modeled as edge cracks in bending, the approximate stress intensity (40) at the tip of a single interdendritic shear band of  $2 \mu\text{m}$  in length would be  $1.9 \text{ MPa}\sqrt{\text{m}}$  at the stress, corresponding to the fatigue limit of  $\sigma_a = 0.3\sigma_{UTS}$ . This is approximately equal to the measured fatigue-crack-growth threshold stress intensity for the

monolithic glass (10, 16) and is consistent with no failure in the BMG composite at  $2 \times 10^7$  cycles. In contrast, for the LM2 glass-matrix composite with a smaller volume fraction of dendrites and interdendritic glass thicknesses of  $\approx 10 \mu\text{m}$  (23, 28), a shear band could grow 5 times larger before arrest by the dendrites. The threshold stress intensity can now be reached at much lower applied stress of  $\sigma_a = (0.3/5^{1/2})\sigma_{UTS} \approx 0.1\sigma_{UTS}$ , as observed experimentally. This presents a simple hypothesis for improving the low fatigue limits in metallic glasses. The characteristic spacing,  $D$ , which separates second-phase inclusions in a glassy matrix (and thereby confines the shear-band length), should be such that  $\alpha\sigma_a D^{1/2} \sim K_{th}$ , where  $K_{th}$  is the critical stress-intensity threshold for fatigue-crack propagation in the monolithic glass and  $\alpha$  is a constant of order unity. Equivalently, one predicts a fatigue limit of  $\sigma_a \sim K_{th}/\alpha D^{1/2}$ . In the absence of any microstructure, as in monolithic BMG, it is clear that fatigue limits will be very low because  $D$  becomes essentially infinitely large.

**Other Considerations.** In addition to the spacing, one might ask whether the microstructural topology of the dendritic phase is also important. This is especially pertinent to in situ glass-matrix composites, because recent studies on La-based BMG–dendrite alloys have shown that the ductility and toughness of these alloys, at both room (41) and elevated (42) temperatures, can be quite different above and below the percolation threshold for the second-phase dendrites. Whereas this may be important for “global” properties such as the resistance to fatigue-crack propagation (and ductility and toughness) where a crack could span many characteristic microstructural dimensions, we doubt whether it would have too much influence on a property such as the fatigue limit, which depends on distinctly “local” phenomena, specifically the initiation and early growth of a micrometer-sized shear-band crack within the glassy phase and its arrest at the glass–dendrite interface.

One might also argue that the fatigue limits of the BMG-composite alloys are much higher than those of the monolithic BMG materials simply because they contain a high fraction of a crystalline (dendritic) phase. However, in similar vein, because the critical event associated with the definition of the fatigue limit is the local arrest of a small crack at the BMG–dendrite interface, the fatigue properties of the dendritic phase itself are far less important than the crack-arresting capability of the interface.

Finally, there are data in the literature, specifically from Liaw and co-workers (43–45), that report extremely high  $\sigma_a$  fatigue limits for several monolithic Zr-based BMG alloys that are as large as  $\approx 0.25\sigma_{UTS}$ , results that are totally inconsistent with fatigue-limit measurements by other investigators (20, 21) on similar alloys that we have quoted in this article. We believe that there are two reasons for this inconsistency. First, as suggested by Schuh et al. (46), the Liaw group’s specimens were machined from relatively small ingots, whereas those used by other investigators (16, 20–22) were machined from cast plates. Although this could have led to differences in free volume and residual stresses due to variations in cooling (15, 47), we do not believe that this factor is that significant. A second, more significant reason is that there is a major difference in the specimen geometries used; Liaw and co-workers (43–45) used a notched cylinder geometry whereas all other investigators have used unnotched rectangular bend bars. For the measurement of material properties, such as fatigue limits, the notched geometry used by Liaw and co-workers is a particularly poor choice, simply because there will always be significant uncertainty in the value of the stress

concentration factor to use to define the fatigue-limit stress.<sup>†</sup> Indeed, after careful analysis of the stress state and final fracture surfaces for the notched specimens of Liaw and co-workers (43, 44), Menzel and Dauskardt (48) concluded that an incorrect stress concentration factor had been used. It is for this reason that we strongly believe that the unsubstantiated and unreasonably high fatigue limits measured by Liaw and his colleagues (43–45) are in error.

**Closure.** In conclusion, our results on the new DH3 alloy highlight the potential of using designed composite microstructures for bulk metallic glass alloys to provide an effective solution, not simply to their low tensile ductility and toughness but also to their characteristically poor stress–life fatigue properties. Provided the characteristic length scales of crack size and microstructure are correctly matched, both to retard the initial extension of small flaws and to prevent single shear-band opening failure, BMG materials can be made with high strength (>1.2 GPa), substantial tensile ductility (>10%), and fatigue limits that exceed those of high-strength steels and aluminum alloys.

## Methods

**Design of Alloys.** The metallic glass-matrix  $Zr_{39.6}Ti_{33.9}Nb_{7.6}Cu_{6.4}Be_{12.5}$  alloys used in this research were prepared in a two-step process. First, ultrasonically cleansed pure elements, with purities 99.5%, were arc-melted under a Ti-gettered argon atmosphere. The ingots were formed by making master ingots of Zr–Nb and then combining those ingots with Ti, Cu, and Be. Ti and Zr crystal bars were used, and other elements were purchased from Alfa Aesar in standard forms. Second, the ingots were placed on a water-cooled Cu boat and heated via induction, with temperature monitored by pyrometer. The second step was used as a way of semisolidly processing the alloys between their solidus and liquidus temperatures. This procedure coarsens the dendrites, produces radio-frequency stirring, and homogenizes the mixture. Sam-

ples were produced with masses up to 35 g and with thicknesses of 10 mm, based on the geometry of the Cu boat. Samples for mechanical testing were machined directly from these ingots.

**Characterization.** Microstructures were characterized using an interference contrast technique on a Axiotech 100 reflected-light microscope (Carl Zeiss MicroImaging) and scanning electron microscopy (SEM) (S-4300SE/N ESEM; Hitachi America) operating in vacuo ( $10^{-4}$  Pa) at a 30-kV excitation voltage in both secondary and back-scattered electron modes. Samples were mechanically wet polished with an increasingly higher finish to a final polish with a 1- $\mu$ m diamond suspension. No etching was performed.

**Stress–Life Experiments.** Fatigue-life ( $S$ – $N$ ) curves were measured over a range of cyclic stresses by cycling  $3 \times 3 \times 50$  mm rectangular beams in four-point bending (tension–tension loading) with an inner loading span,  $S_1$ , and outer span,  $S_2$ , of 15 and 30 mm, respectively, in a computer-controlled, servo-hydraulic MTS 810 mechanical testing machine (MTS Corporation). The corners of the beams were slightly rounded to reduce any stress concentration along the beam edges, and they were then polished with diamond paste to a 1- $\mu$ m finish on the tensile surface before testing. Testing was conducted in room air under load control with a frequency of 25 Hz (sine wave) and a constant load ratio (ratio of minimum to maximum load,  $R = P_{\min}/P_{\max}$ ) of 0.1. Stresses were calculated at the tensile surface within the inner span using the simple beam mechanics theory:

$$\sigma = \frac{3P(S_2 - S_1)}{2BW^2}, \quad [1]$$

where  $P$  is the applied load,  $B$  is the specimen thickness, and  $W$  is the specimen height. Beams were tested at maximum stresses ranging from 560 to 1,150 MPa (just below the ultimate tensile strength). Tests were terminated in cases where failure had not occurred after  $2 \times 10^7$  cycles ( $\sim 9$  days at 25 Hz). Fracture surfaces of selected beams were examined after failure by both optical microscopy and SEM to discern the origin and mechanisms of failure. The stress–life fatigue data ( $S$ – $N$ ), shown in Fig. 2, are presented in terms of the number of cycles to failure,  $N_f$ , and stress amplitude ( $\sigma_a = \frac{1}{2}\Delta\sigma = \frac{1}{2}[\sigma_{\min} - \sigma_{\max}]$ ) normalized by the ultimate tensile strength of the material ( $\sigma_{UTS}$ ), where  $\Delta\sigma$  is the stress range and  $\sigma_{\max}$  and  $\sigma_{\min}$  correspond, respectively, to the maximum and minimum values of the applied loading cycle.

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<sup>†</sup>The stress concentration factor under fatigue conditions is invariably not the elastic stress concentration factor,  $k_t$ , which can be well defined by the geometry and loading conditions. In fatigue, an effective stress concentration factor,  $k_f$ , must be used that will be less than or equal to  $k_t$ , depending upon the material and size of the notch. Because the value of  $k_f$  cannot be predicted or even calculated, it must be defined from experimental data in terms of the ratio of the alternating stress to give a specific life in an unnotched fatigue test divided by the corresponding alternating stress to give the same life in a notched test, a procedure that was not utilized by Liaw and co-workers (43–45).

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