Why Ductile Fracture Mechanics?[1]

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Until recently, the engineering application of fracture mechanics has been specific to a description of macroscopic fracture behavior in components and structural parts which remain nominally elastic under loading. While this approach, termed linear elastic fracture mechanics, has been found to be invaluable for the continuum analysis of crack growth in brittle and high strength materials, it is clearly inappropriate for characterizing failure in lower strength ductile alloys where extensive inelastic deformation precedes and accompanies crack initiation and subsequent propagation. Accordingly, much effort has been devoted in recent years toward the development of nonlinear or ductile fracture mechanics methodology to characterize fracture behavior under elastic/plastic conditions; an effort which has been principally motivated by problems in nuclear industry. In this paper, the concepts of ductile (elastic/plastic) fracture mechanics are introduced and applied to the problem of both stationary and nonstationary cracks. Specifically, the limitations inherent in this approach are defined, together with a description of the microstructural considerations and applications relevant to the failure of ductile materials by fracture, fatigue, and creep.

Introduction

Since its earliest origins in the 1950's, the development of fracture mechanics has presented both the materials scientist and the mechanical engineer with a powerful means to quantitatively describe the *macroscopic* fracture behavior of solids. On the one hand, the use of fracture mechanics has permitted the materials scientist to perform meaningful comparisons between different materials on the role of alloy composition, microstructure, stress-state, crack size, etc. in influencing such processes as monotonic fracture, fatigue crack propagation, and environmentally-affected crack growth. In fact, it has provided a continuum-mechanics framework for the presentation of laboratory test data in order to quantitatively evaluate the fracture properties of materials. To the engineer, on the other hand, fracture mechanics has provided methodology to utilize such laboratory data (which are generally derived from small samples) to quantitatively predict the structural integrity of larger components in service, and to aid in the analysis of service failures. Further, this is achieved without any recourse to formulating microstructural models of the complex fracture processes involved. The essential premise in this approach has been the realization that all materials contain defects and incipient flaws, such that the expected life-time of a given component can be considered in terms of the time required to propagate the largest undetected crack (estimated from proof testing or through non-destructive evaluation) to some critical size (estimated from the fracture toughness, limit load or design requirements). This approach, known as defect-tolerant design, is now in widespread use, particularly

for safety-critical structures such as are encountered in nuclear and aerospace applications.

To date, the engineering applications of fracture mechanics have centered around a description of macroscopic fracture behavior in components and structural parts which remain nominally elastic. Such linear elastic fracture mechanics, however, while proving to be invaluable for the continuum analysis of crack growth in brittle and high strength materials, becomes inappropriate when applied to the description of failure in lower strength ductile materials where extensive inelasticity precedes and accompanies fracture. To meet this need, much analytical and experimental effort has been devoted in recent years toward development of nonlinear or ductile fracture mechanics to characterize crack growth where fracture initiation and subsequent crack advance occurs under elastic/plastic conditions.

It is the objective of this paper to review the concepts of ductile (elastic/plastic) fracture mechanics, as applied to both stationary and nonstationary cracks, and to highlight the inherent limitations of its use. Furthermore, the microstructural considerations and applications of this approach are described with respect to the failure of ductile alloys by fracture, fatigue and creep.

Linear Elastic Fracture Mechanics

The essential features of fracture mechanics begin with characterizing the stress and deformation fields, local to the region at a crack tip. This is achieved principally through the use of asymptotic continuum mechanics analyses where the functional form of the local singular field is determined within a scalar amplitude factor whose magnitude is calculated from a complete analysis of the applied loading and geometry. The best known example of this approach is for the linear elastic behavior of a stationary crack subjected to tensile (Mode I) opening (Fig. 1), where the local crack tip
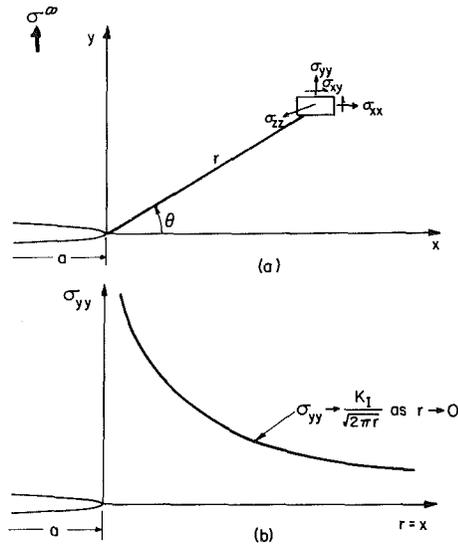
Fig. 1 Schematic representation of a half-crack, length a, subjected to a Mode I remotely-applied stress σ^∞, showing the linear elastic distribution of the local tensile stress (σ_{yy}) directly ahead of the crack

stresses (σ_{ij}) can be characterized in terms of the K_I singular field [1,2]:

$$\sigma_{ij}(r,\theta) \rightarrow \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \quad \text{as } r \rightarrow 0, \qquad (1)$$

where K_I is the Mode I stress intensity factor, r the distance ahead of the crack tip, θ the polar angle measured from the crack plane, and f_{ij} a dimensionless function of θ. Similar expressions exist for cracks subjected to pure shear (Mode II) and anti-plane strain (Mode III). Provided this asymptotic field can be considered to "dominate" the local crack tip vicinity over a region which is large compared to the scale of the microstructural deformation and fracture events involved, then the scalar amplitude factor K_I can be considered as a single, configuration-independent parameter which uniquely and autonomously characterizes the local stress field ahead of a linear elastic crack and can be used there as a correlator of crack extension. Although undetermined from the asymptotic analysis, K_I can be computed from the overall geometry and applied loading conditions, and solutions for K_I applicable to a wide variety of situations are now tabulated in handbooks [e.g. 3]. For example, for the case of an internal crack of length $2a$ in an infinite body subjected to a remotely applied tensile stress σ^∞, K_I is simply given by

$$K_I = \sigma^\infty \sqrt{\pi a}. \qquad (2)$$

Thus for linear elastic conditions, crack tip fields can be considered to be unique to within a scalar factor K_I, such that K_I constitutes a single-parameter crack driving force for crack advance. For the monotonic loading of stationary cracks, this approach has been applied to characterize the onset of brittle fracture, where for plane strain conditions $K_I = K_{IC}$, the fracture toughness [4], and to estimate the onset of crack instability in plane stress through the use of K_I-resistance curves [5]. Furthermore, for sub-critical crack growth, K_I has been used to correlate rates of crack growth both for environmentally-assisted fracture (stress corrosion, hydrogen embrittlement, etc.) and in fatigue (through expressions of the form $da/dN = C\Delta K^m$ [6]. The essence of this approach and in fact the reason why it can be successfully applied to such a wide range of fracture behavior is that the asymptotic continuum mechanics characterization does not necessitate detailed quantitative microscopic models to be known for the individual fracture events. In view of the complexity of these processes on the microstructural scale, this must be regarded as fortunate, at least, for a macroscopic description of fracture [8].
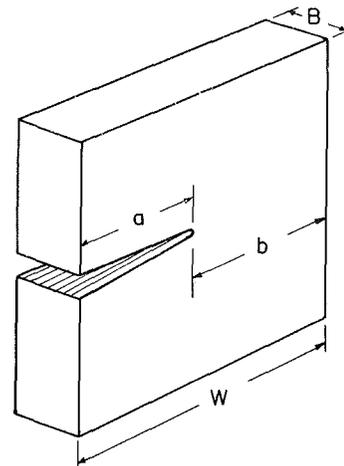

Fig. 2 Idealized geometry showing definition of thickness (B) and inplane dimensions of crack length (a) and ligament depth (b = W − a)

Naturally, there are limitations inherent in this approach. First, equation (1) ignores all but first order terms, such that a K_I characterization of crack tip fields is only relevant as r tends to zero, i.e., K_I cannot be taken as a correlator of crack extension if, for example, the scale of microscopic fracture events (the so-called characteristic or microstructurally-significant dimension) is as large as the crack length. However, as r tends to zero, stresses become infinite for the linear elastic analysis. In reality, of course, such stresses are limited by local crack tip yielding, which occurs over a region ahead of the crack tip known as the plastic zone size r_y. Calculations of the extent of this region vary depending upon the mode of applied loading and the geomety of the body [7, 8] but a rough estimate for r_y can be taken as

$$r_y \approx \frac{1}{2\pi} \left(\frac{K_I}{\sigma_0} \right)^2, \qquad (3)$$

where σ_0 is the yield strength of the material. Thus, although the linear elastic stress distribution, characterized by the K_I-field (equation (1)), is only valid close to the crack tip (i.e., as $r \rightarrow 0$), it is violated there over a dimension of the order of $(K_I/\sigma_0)^2$, i.e., the asymptotic solution is most accurate where it is least relevant! However, provided the extent of local plasticity is small compared with the extent of the K_I-field, which itself is small compared to overall dimensions of the body (including the crack length), the plastic zone can be considered as merely a small perturbation in the linear elastic field and K_I crack tip dominance can be preserved. For an idealized geometry (Fig. 2), this situation, known as small-scale yielding, appears to be met when the plastic zone is of the order of 15 times smaller than the in-plane dimensions of crack length (a) and ligament depth (b). Additionally, where the K_I approach is used to define a single-valued characterization of toughness, i.e., for the onset of brittle fracture at $K_I = K_{IC}$, the requirement of plane strain must also be met such that the plastic zone must be approximately 15 times smaller than the out-of-plane dimension of thickness B. These limitations form the basis for the minimum test-piece size requirements of the ASTM E-399 Standard for K_{IC} determination [4], i.e., that

$$a, B, b \geq 2.5 \left(\frac{K_{IC}}{\sigma_0} \right)^2. \qquad (4)$$

Such limiting size requirements for the use of linear elastic fracture mechanics actually present few practical difficulties for most higher strength or brittle materials (Table 1). For example, valid K_{IC} measurements can be made for maraging steels with test specimens larger than approximately 14 mm, and for tungsten carbide with specimens thicker than 0.3 mm.

Table 1 Approximate limiting size requirements for characterization by linear elastic fracture mechanics in different materials

Material	σ_0 (MPa)	K_{IC} (MPa√m)	r_y (μm)	Limiting size (mm)
4340, 200°C temper	1700	60	200	3 (~0.1 in.)
Maraging Steel	1450	110	920	14 (~0.5 in.)
A533B-1	500	245	4×10^4	600 (~2 ft)
7075-T651	515	28	470	7 (~0.3 in.)
2024-T351	370	35	1420	22 (~1 in.)
Ti-6Al-4V	850	120	3170	50 (~2 in.)
Tungsten Carbide	900	10	20	0.3 (~3 mils)
Polycarbonate	70	3	290	5 (~0.2 in.)

However, characterizing the fracture toughness of a lower-strength ductile material, such as A533B-1 nuclear pressure vessel steel, would necessitate the use of a test-piece 2 foot thick containing a similar sized fatigue precrack! While such jumbo-sized specimens have been tested in a few instances [9], the cost associated with large-scale testing of this type is generally totally prohibitive. Further, in the case of nuclear materials where the toughness of irradiated samples is required, such test-pieces simply could not be utilized.

The need, therefore, exists for a means to reliably measure the fracture toughness of such lower strength ductile materials as nuclear pressure vessel steel in laboratory-size test-pieces, where fracture is accompanied by extensive deformation (large-scale yielding), and to use this information to predict failure in the much larger section sizes encountered in service (where conditions of small-scale yielding may apply). Additionally, an extension of the linear elastic characterization is required for the macroscopic fracture analysis of such problems as creep crack growth, fatigue crack propagation at high stress intensities and the growth of small cracks; all instances where the extent of local crack tip plasticity is comparable with crack length and overall geometric dimensions. Such an extension has been provided by the development of ductile (elastic/plastic) fracture mechanics.

Ductile (Elastic/Plastic) Fracture Mechanics

As shown above, the restriction of small-scale yielding places a severe limitation on the application of linear elastic fracture mechanics, a restriction which effectively excludes lower strength ductile materials. Whereas several approaches have been suggested over the years to extend linear elastic fracture mechanics to situations where plastic zones are larger (e.g. for plane stress [5]), K_I-field crack tip solutions in general cannot be utilized for large-scale yielding conditions and elastic/plastic solutions must be sought. Such solutions were first proposed in 1968 by Hutchinson, Rice and Rosengren [10, 11] for power-hardening solids ($\sigma \propto \epsilon_{plastic}^n$) under symmetric opening loads. The HRR singularity, as it has become known, yields an asymptotic form of the crack tip stress and strain fields which, in the limit as $r \to 0$, gives

$$\sigma_{ij}(r,\theta) \to \left(\frac{EJ}{\sigma_0^2 r} \right)^{n/(n+1)} \cdot \sigma_0 f_{ij}(\theta,n), \tag{5}$$

$$\epsilon_{ij}(r,\theta) \to \left(\frac{EJ}{\sigma_0^2 r} \right)^{1/(n+1)} \cdot g_{ij}(\theta,n), \tag{6}$$

where σ_0 is the yield or flow strength, n the work hardening exponent, E the elastic modulus, and f_{ij} and g_{ij} are universal functions of their arguments dependent upon whether plane strain or plane stress is assumed. The amplitude of the asymptotic field J is the so-called J-integral, introduced by Rice and Cherepanov [12, 13], which can be defined for any closed contour around a crack tip as

$$J = \int_\Gamma W dy - T_i \frac{\partial u_i}{\partial x} ds, \tag{7}$$

where T is the traction vector perpendicular to Γ and W is the
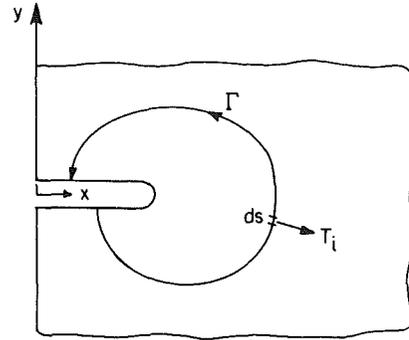

Fig. 3 Showing contour Γ drawn counter-clockwise around crack tip in definition of J-integral
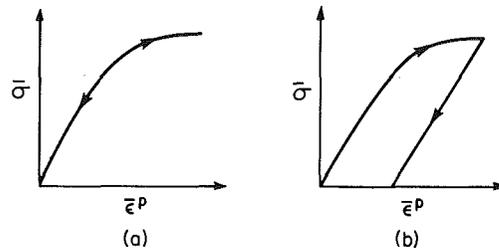

Fig. 4 Idealized constitutive behavior, of equivalent stress $\bar{\sigma}$ as a function of equivalent plastic strain $\bar{\epsilon}_p$, for (a) nonlinear elastic material conforming to deformation plasticity theory, and (b) incrementally-plastic material conforming to flow theory of plasticity

strain energy density, as shown in Fig. 3. It can be shown that the J-integral is precisely path-independent for nonlinear elastic materials conforming to deformation theory plasticity (Fig. 4(a)) and substantially path-independent for numerical solutions of incrementally plastic materials conforming to flow theory (Fig. 4(b)) [8]. Furthermore, by choosing the contour Γ to fall within the region dominated by the K_I-field for small-scale yielding, J can be directly related to the strain energy release rate G and hence to the stress intensity K_I for linear elastic behavior [7], i.e.,

$$J = G = K_I^2 / E', \text{ (linear elastic)} \tag{8}$$

where $E' = E$ for plane stress and $E/(1 - \nu^2)$ for plane strain.

Examination of equations (5) and (6) reveals that in directly analogous fashion to the function of K_I in defining the amplitude of linear elastic crack tip fields (equation (1)), the HRR singularity yields elastic/plastic crack tip singular fields which are unique (for a strain hardening material) to within a scalar amplitude factor J. Once again, provided J-dominance is assured over regions ahead of the crack tip comparable with the scale of the microstructural deformation and fracture events involved, J, like K_I, can be used as a correlator of crack extension only now for elastic/plastic conditions. Furthermore, by recognizing the equivalence of J and G in linear elasticity, values of the stress intensity K_I can be determined from J for small-scale yielding through the use of equation (8).

At this point it is worth noting that from equations (5) and
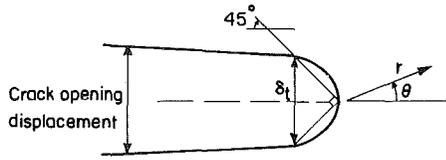
Fig. 5 Definition of the crack tip opening displacement (CTOD), δ_t

(6), the opening of the crack faces varies at $r \to 0$ as $r^{n/(n+1)}$. This separation can be used to define the crack tip opening displacement (CTOD) δ_t as the opening where 45 deg lines intercept the crack faces (Fig. 5) such that

$$\delta_t = d(\epsilon_0, n) J/\sigma_0, \qquad (9)$$

where d is a proportionality factor dependent upon the yield strain ϵ_0 and work hardening exponent n, which varies for plane stress as opposed to plane strain. From Shih's numerical computations [14], d has been found to vary from 1 for $n = 1$ to 0.4 for $n = 0.3$ in plane stress and from 0.8 for $n = 1$ to 0.3 for $n = 0.3$ in plane strain. Similar to J, δ_t can also be considered as a measure of the intensity of the elastic/plastic crack tip fields, yet unlike J, it perhaps offers more physical insight since it can be more readily related to the physical crack tip failure processes involved [15].

As in linear elastic fracture mechanics, the J or CTOD approach has been applied to numerous modes of fracture behavior. For stationary cracks under monotonically increasing proportional loading in plane strain, J has been used to characterize the initiation of cracking (at $J = J_{IC}$ or at $\delta_t = \delta_i$), whereas for nonstationary cracks subsequent crack growth has been analyzed with J-resistance curves using parameters such as dJ/da (the slope of the J-resistance curve), T (the tearing modulus), and CTOA (the crack tip opening angle, $d\delta_t/da$) [16]. Other applications have been the use of ΔJ, the cyclic range of J, for characterizing the rate of elastic/plastic fatigue crack propagation, and J or C^*, the rate-dependent analogue of J, for creep crack growth rates. These applications are described in more detail below.

There are several factors which must be considered, however, before the use of J (or δ_t) can be contemplated for the above mentioned applications. First, the underlying assumption in deriving the HRR solutions (equations (5) and (6)) and the energy release rate definition of J (equation (7)) are that material behavior conforms to the deformation theory of plasticity (i.e., the material is a nonlinear elastic solid as in Fig. 4(a)). For a stationary crack subject to a monotonically increasing load, where plastic loading will not depart radically from proportionality, this is a good approximation. However, for growing cracks where regions of elastic unloading and nonproportional plastic flow will be embedded in the J-dominated field, behavior is not properly modelled by deformation theory, and this poses certain restrictions to the J characterization for large-scale yielding as discussed below [17]. Second, for J or δ_t to be utilized as a single, configuration-independent parameter to characterize crack extension, the HRR fields must dominate over a region ahead of the crack tip which is large compared to the scale of the microstructural deformation and fracture events involved. Since this fracture process zone is of the order of the blunted crack opening, i.e., the CTOD, the radius of the HRR field (i.e., the zone of dominance R) must be large compared to δ_t. This, like the conditions for K_I-dominance (small-scale yielding) and valid K_{IC} measurement in the linear elastic analysis, implies that certain specimen size requirements must be met for the J analysis to be relevant. Unfortunately, unlike the linear elastic case, these size limitations (i.e., the region of J-dominance) can vary markedly in different specimen geometries. In this regard it is worth remembering that crack tip fields for rigid/perfectly plastic bodies under fully yielding conditions are not unique, implying that there can be no
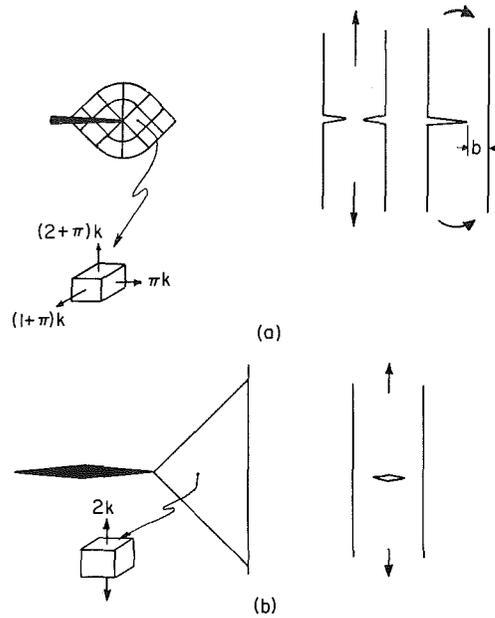

Fig. 6 Fully plastic plane strain slip-line fields for rigid/perfectly plastic solids for (a) deep edge-cracked bend and deep double-edge-cracked tension plates (Prandtl field), and (b) center-cracked tension plate. k = shear yield stress = $\sigma_0/\sqrt{3}$.

unique, configuration-independent parameter (i.e., J or anything else) which is a measure of crack tip deformation and extension in this limit. As noted by McClintock [18], the plane strain slip-line field for a fully-yielded edge-cracked plate in bending has a fundamentally different near-tip stress and strain field compared to the center-cracked plate in tension (Fig. 6). The former case, which is essentially the Prandtl field, develops high triaxial and normal stress ahead of the tip, with r^{-1} singular shear strains in the fan above and below, whereas in the latter case only modest triaxiality occurs ahead of the tip, but intense shear strains develop on planes at 45 deg to the crack. Rationalizing such nonunique fully plastic solutions with our originally stated concept of a unique HRR field at the crack tip requires that some strain hardening must exist for J-controlled crack extension. However, the region of relative dominance of the HRR singularity for strain hardening materials will correspondingly be significantly smaller for the center-cracked plate in tension compared with the edge-cracked plate in bending. Finite strain, finite element calculations by McMeeking and Parks [19] have quantitatively estimated these size limitations for a single parameter J characterization, in terms of the ligament dimension b, as

$$b > 25 \frac{J}{\sigma_0}, \text{ for edge-cracked bend specimen} \qquad (10)$$

and

$$b > 200 \frac{J}{\sigma_0}, \text{ for center-cracked tension specimen} \qquad (11)$$

for materials of moderately low strain hardening ($n = 0.1$). It is immediately apparent from these calculations that the center-cracked plate in tension is subject to much more stringent size requirements, which place a severe limitation on the applicability of a single parameter fracture characterization to such cracked configurations.

Applications of Ductile Fracture Mechanics

A. Crack Initiation (Stationary Cracks). The potential application of elastic/plastic fracture mechanics, in particular the use of J, to characterize the onset of crack extension in ductile materials, i.e., to determine the fracture toughness

under large-scale yielding conditions, was first developed by Begley and Landes [20, 21]. On the premise that, using the HRR singularity, J uniquely and autonomously characterizes the crack tip stress and strain fields around a stationary crack in a strain hardening material, they proposed that for plane strain conditions, at the initiation of crack growth, J would exceed some critical value J_{IC}. Thus, by determining J_{IC} in a small-specimen large-scale yielding test, the fracture toughness K_{IC} (for small-scale yielding) could then be computed using the J-K_I equivalence stated in equation (8). The advantages of such a test can be readily appreciated by comparing the test-piece size requirements with those formerly stated for valid K_{IC} measurement (equation (4)). By considering again, A533B nuclear pressure vessel steel with a compact tension geometry (essentially equivalent to the Prandtl field), the valid small-scale yielding K_{IC} test requires a 2 ft thick specimen, whereas the large-scale yielding J_{IC} test merely requires the thickness and ligament depth to exceed 25 J_{IC}/σ_0 (from equation (10)). Using the values quoted in Table 1, this means that the fracture toughness can be measured in A533B steel with only a 12 mm (1/2 in.) specimen, which is clearly a practical size for standard laboratory test measurements. It should be noted here, however, that had a center-cracked tension specimen been employed, the more stringent size limitations [19] of this geometry for J-dominance (equation (11)) would have necessitated the use of a 100 mm specimen (i.e., $B,b > 200 J_{IC}/\sigma_0$).

Test methods to determine the fracture toughness with J_{IC} measurements have become standardized and involve the determination of the value of J at crack initiation using the J-resistance curve (Fig. 7) [22]. Using a series of identical test-pieces[1] (the multispecimen technique) or a single test-piece[1] with an independent means of monitoring crack growth (i.e., using unloading compliance), values of J corresponding to different amounts of crack extension (Δa) are plotted to construct the resistance curve $J_R(\Delta a)$. The value of J_{IC} at crack initiation is then found, by extrapolating the linear portion of this curve to the point of zero crack extension, characterized by the so-called blunting line defined as

$$J = 2\sigma_0 \Delta a. \tag{12}$$

Similar to K_I-solutions, solutions for J in a wide variety of loading and cracked configurations can be obtained from handbooks [23].

Analogous methods for determining the fracture toughness under large-scale yielding conditions have also been developed using the CTOD concept [24]. Although crack initiation δ_i values are physically more appealing in terms of the relationship of macroscopic toughness parameters to the actual microscopic failure events involved, the crack tip opening displacement is more difficult to measure and interpret, and is generally not favored in this country.

B. Crack Growth (Nonstationary Cracks). The extension of elastic/plastic fracture mechanics to the case of growing cracks is considerably less developed in view of the fact that near-tip stress and strain fields for the nonstationary flaw are far more complex. For example, crack growth will involve elastic unloading and nonproportional plastic loading, both of which are inadequately described by the deformation theory of plasticity on which J is based [16]. However, following the analysis of Hutchinson and Paris [17], it is apparent that under restricted circumstances, the concept of J-controlled growth based on the $J_R(\Delta a)$ resistance curve can be used. Figure 8 shows a schematic representation of the near-tip conditions for a growing crack [16]. Regions of elastic unloading (comparable with the scale of crack advance

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[1] To prevent tunnelling of crack growth at the center of the specimen, such test-pieces may be side-grooved to a depth of the order of 20 percent of the thickness.
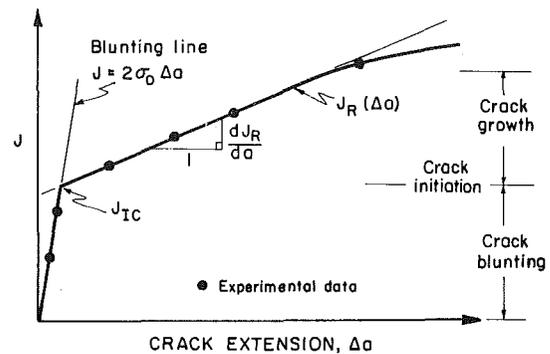

Fig. 7 $J_R(\Delta a)$ resistance curve, showing definition of J_{IC} at initiation of crack growth
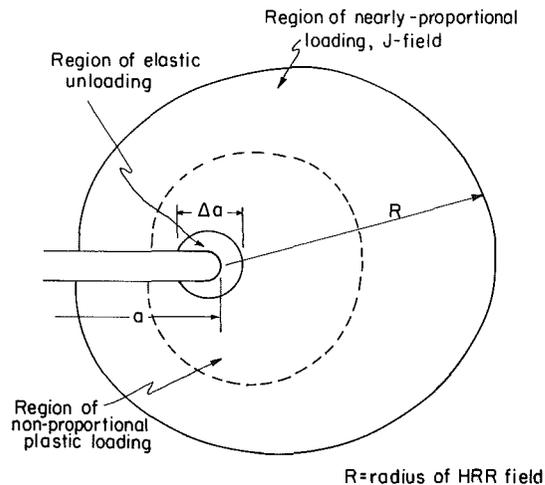

Fig. 8   Schematic representation of the near-tip conditions for a nonstationary crack relevant to the definition of J-controlled growth

Δa) and nonproportional loading are embedded within the HRR J-controlled singularity field of radius R. The argument for J-controlled crack extension relies on the fact that provided these regions are small compared to the radius of the HRR field, then the singularity field can be said to be controlling. This is essentially the same concept used in linear elastic analysis where a region of plastic behavior, i.e., the plastic zone, is considered to be embedded in, and controlled by, the K_I singularity field. The two conditions for J-controlled growth are thus that the region of elastic unloading is small, i.e.,

$$\Delta a << R, \tag{13}$$

and that J increases sufficiently rapidly with crack extension such that the region of non-proportionality is small, which can be stated as [17]:

$$\omega \equiv \frac{b}{J_{IC}}\left(\frac{dJ_R}{da}\right) >> 1. \tag{14}$$

Numerical calculations by Shih and co-workers [23, 25] interpret these requirements for J-controlled growth as $\omega \gtrsim 10$ for Prandtl field geometries and $\omega \gtrsim 100$ for center-cracked tension geometries. This means that the concept of J-controlled crack extension of a nonstationary crack in plane strain (i.e., $B > b$) is valid only for crack growth corresponding to 6 percent of the ligament ($\Delta a < 0.06b$) in a compact tension geometry. Thus, using a typical precracked 25 mm thick 1T compact specimen, only the first 1.5-2 mm of crack extension can be taken as J-controlled. Furthermore, for the center-cracked tension configuration, this requirement is even more restrictive and corresponds to crack growth over only 1 percent of the ligament (i.e., $\Delta a < 0.016b$, which corresponds to roughly 0.5 mm for a 25 mm ligament).

CRACK GEOMETRY	SLIP-LINE FIELD	Nominal δ_i (μm) J_{IC} (kJ/m^2) (HY80 steel)	
Double-edged cracked (W=10b)		90	147
Three-point bend and compact tension		170	190
Double-edged cracked (W=4b)		302	338
Single edge-cracked tension		450	504
Single edge-cracked tension (center ligament loaded)		900	570
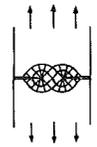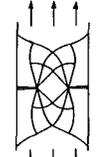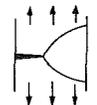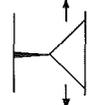
Fig. 9 Nominal δ_i and J_{IC} values determined for HY80 steel for a variety of crack configurations. Data from Hancock and Cowling (1980).

Despite these stringent size limitations for J-controlled crack advance, several criteria have been proposed to characterize the toughness and stability of the extending crack based or the J_R (Δa) resistance curve (Fig. 7). Paris and co-workers [26], for example, have proposed an analysis of crack instability similar to the linear elastic resistance curve concept for plane stress crack extension. By characterizing the tearing resistance of a material in terms of the nondimensional slope of the J_R (Δa) resistance curve, i.e.,

$$T_R = \frac{E}{\sigma_0^2} \cdot \frac{dJ_R}{da}, \tag{15}$$

where T_R is known as the tearing modulus, crack instability is achieved when the tearing force ($T = (E/\sigma_0^2)\ \partial J/\partial a$) exceeds T_R. Using a variety of specimen geometries in several widely different materials, some success has been achieved in correlating crack growth and instability using this concept. Analogous procedures have been developed using the slope of the CTOD-resistance curve, where $d\delta_t/da$ is equivalent to the crack tip opening angle (CTOA)[23, 27].

C. Other Applications of Ductile Fracture Mechanics. The use of the J concept to characterize crack extension has also been applied to the problem of fatigue crack propagation, where the parameter utilized to correlate rates of crack growth (da/dN) is now taken to be ΔJ, the cyclic range of J for each stress reversal [28]. Similar to the case of monotonic crack extension of a nonstationary flaw described above, this application again appears to violate the basic assumption of deformation plasticity theory that stress is proportional to current plastic strain. However, by recognizing that constitutive laws for cyclic plasticity (i.e., the cyclic stress-strain curve) can be considered in terms of *stable* hysteresis loops, and that such loops can be shifted to a common origin after each half cycle, the criterion of stress proportional to current plastic strain can be effectively achieved. Some success has been achieved with this elastic/plastic fracture mechanics approach in correlating fatigue crack growth rates at high stress intensities [28], for

small cracks [29] and for crack extension in Mode III (antiplane strain) [30], all instances where the extent of local crack tip plasticity (i.e., the plastic zone size) is too large to permit a small-scale yielding characterization in terms of ΔK, the cyclic stress intensity range.

Elastic/plastic fracture mechanics has also been applied to the problem of creep crack growth at elevated temperatures, where now the asymptotic crack tip fields can be scaled in terms of C^*, the rate-dependent or viscous analogue of J [31]. Fundamentally, interpretation is far more complex in this case as the strength and region of dominance of the local HRR fields are continuously changing with time, and further such fields must be matched with additional K_I- and time-dependent creep deformation fields [8, 31, 32]. However, recent numerical and experimental studies have shown that provided due attention is given to determining the dominant field specific to a given instant in time, such elastic/plastic fracture mechanics analysis can provide a useful macroscopic characterization of crack extension in a power-law creeping solid [31, 32].

Relationship to Microscopic Fracture Modelling

One of the main advantages of fracture mechanics analysis is that it effectively correlates the macroscopic aspects of crack initiation and growth without recourse to developing microscopic models for the local fracture processes which themselves must depend upon the nature of the microstructure and the local crack tip stress and deformation histories. However, for a complete understanding of fracture such microstructural initiation and growth criteria must be defined and related to the macroscopic continuum analyses. In a few simplified cases, this has been achieved. For example, for slip-initiated transgranular cleavage fracture in ferritic steels, Ritchie, Knott and Rice [33] have shown that the onset of brittle crack propagation at $K_I = K_{IC}$ is consistent with the local tensile opening stress (σ_{yy}), directly ahead of the crack, exceeding a local fracture stress (σ_f^*) over a microstructurally-significant characteristic distance. In mild steels, this distance appeared to be of the order of two grain diameters, although other size-scales have been found when the analysis is applied to other materials [34]. Similarly, a stress-modified critical strain criterion has been found for crack initiation by microvoid coalescence where, at $J = J_{IC}$, the local equivalent plastic strain must exceed some critical fracture strain or ductility (specific to the relevant stress-state) over a characteristic distance comparable with the spacing of the void-initiating particles [35]. Crack extension in Mode III for elastic/perfectly plastic materials has been similarly analyzed in terms of a total shear strain being exceeded over the extent of the plastic zone size [36], whereas the more complex calculations for Mode I crack extension require a critical crack opening displacement to be reached at some fixed microstructural distance behind the growing crack tip [37]. Modelling studies such as these represent the very heart of the understanding of fracture in that they seek to unify microscopic failure mechanisms and the role of microstructure with the continuum asymptotic crack tip stress and strain fields and the macroscopic fracture criteria [33–39]. It is only by such a complete understanding that one can fully proceed from the fundamental alloy design of materials with superior fracture resistance to the engineering predictions of when such materials will fail in service.

Concluding Remarks

In this paper, an attempt has been made to briefly review the extension of linear elastic fracture mechanics to the analysis of failure under elastic/plastic and fully plastic conditions. In view of the very restrictive size requirements for linear elastic fracture characterization in lower strength,

ductile materials, the use of such nonlinear (elastic/plastic) fracture mechanics for these alloys can clearly provide a significant practical payoff. In the nuclear industry, for example, the ability to reliably measure the fracture toughness of pressure vessel steels in laboratory-sized samples, instead of testing 2-ft thick specimens, has saved substantial sums of money, has provided a basis for surveillance specimens, and has enabled a proper characterization of the role of neutron irradiation to be determined. Furthermore, the extension of the analysis for nonstationary cracks may allow future fracture design to be somewhat less conservative in that some amount of stable crack growth can be tolerated. Similar analyses of crack extension by fatigue and creep appear equally promising.

However, although one can feel comfortable about measuring the fracture toughness and subsequent stable crack advance in laboratory compact samples in terms of J, the application of this information to cracked-configurations in service requires far more care than with linear elastic analysis. First, J_{IC} and J-controlled crack growth data pertain specifically to crack advance in plane strain under large-scale yielding conditions. Application of such data, to say, fusion first wall structures where size-scales are small, i.e., in the ~ 2–10 mm range for lower strength ferritic or austenitic stainless steels, may not be appropriate [30]. In this instance, an appreciation of plastic collapse loads and plane stress analyses may be far more relevant. Second, the application of J analysis to be problems of shallow or part-through cracks [41] and to noncoplanar cracks [8] is still largely undeveloped, and yet such configurations are regularly encountered in service. Third, there is the problem of the differing size requirements for J-dominance between various crack geometries and the fact that in the limit of fully plastic failure in nonhardening materials, crack tip stress and deformation fields are simply not unique. In this regard, it is pertinent to note the recent experimental results of Hancock and Cowling [42] on quenched and tempered steels similar to HY80. Using six different cracked configurations of varying degrees of constraint, they found nominal J_{IC} values for crack initiation ranging from ~ 147 to 570 kJ/m^2, despite their claim that the generally accepted size requirements had been met (Fig. 9). Clearly, the size limitations for elastic/plastic fracture mechanics analysis in non-Prandtl field geometries are of extreme importance, and may mean that, for certain configurations, the requirements of size for J-field analysis may be no less restrictive than for the K_I-field solutions. This is particularly relevant for materials of very low strain hardening, a situation which is often the case for highly irradiated alloys. It is clear that for such applications, the use of a single, configuration-independent parameter, such as J, to characterize fracture initiation and crack growth must be viewed with some caution.

Acknowledgments

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