J. Mech. Phys. Solids, 1973, Vol. 21, pp. 395 to 410. Pergamon Press. Printed in Great Britain.

ON THE RELATIONSHIP BETWEEN CRITICAL TENSILE STRESS AND FRACTURE TOUGHNESS IN MILD STEEL

By R. O. Ritchie, J. F. Knott
Department of Metallurgy and Materials Science, Cambridge University

and J. R. Rice
Division of Engineering, Brown University, Providence, R.I.

(*Received 5th April* 1973)

Summary

An analysis is presented which relates the critical value of tensile stress (σ_f) for unstable cleavage fracture to the fracture toughness (K_{Ic}) for a high-nitrogen mild steel under plane strain conditions. The correlation is based on (i) the model for cleavage cracking developed by E. Smith and (ii) accurate plastic–elastic solutions for the stress distributions ahead of a sharp crack derived by J. R. Rice and co-workers. Unstable fracture is found to be consistent with the attainment of a stress intensification close to the tip such that the maximum principal stress σ_{yy} exceeds σ_f over a characteristic distance, determined as twice the grain size. The model is seen to predict the experimentally determined variation of K_{Ic} with temperature over the range -150 to -75°C from a knowledge of the yield stress and hardening properties. It is further shown that the onset of fibrous fracture ahead of the tip can be deduced from the position of the maximum achievable stress intensification. The relationship between the model for fracture ahead of a sharp crack, and that ahead of a rounded notch, is discussed in detail.

Nomenclature

E	Young's modulus
σ_0	flow stress
σ_y	uniaxial yield stress in tension
σ_u	ultimate tensile stress
k	yield stress in pure shear
σ_{yy}	longitudinal tensile stress at notch tip (maximum principal stress)
$\sigma_{yy}^{\max}$	maximum value of σ_{yy}
σ_f	critical tensile stress for unstable cleavage
σ_{nom}	nominal bending stress
$R = \sigma_{yy}/\sigma_0$	stress intensification
$R_{\max} = \sigma_{yy}^{\max}/\sigma_0$	maximum value of R
K	linear elastic stress intensity factor (mode I opening)
K_Q	experimental value of critical K
K_R	K_Q corrected for size of plastic zone
K_{Ic}	plane strain fracture toughness
$K_{Ic}(\rho)$	value of K_{Ic} for fracture ahead of notch of root radius ρ
a	crack length

B	test-piece thickness
W	test-piece width
θ	included angle of notch
α	$\frac{1}{2}\pi - \frac{1}{2}\theta$
d	grain diameter
ρ	notch root radius
ρ_0	'limiting' notch root radius
X	distance below notch tip of material point before deformation
r_y	radius of plastic zone
r_y'	value of r_y where $R = R_{max}$
r_c	critical value of r_y at fracture
L	applied load
L_{GY}	applied load at general yield
$M = \frac{1}{2}LW$	applied bending moment
n	work-hardening coefficient
Y	calibration factor in expression for K
T_T	fibrous/cleavage transition temperature
T_{GY}	temperature at which fracture is coincident with general yield

1. INTRODUCTION

IT IS generally recognized that the plane strain cleavage fracture of mild steel at cryogenic temperatures can be conveniently described in terms of a critical stress criterion for failure. Provided that fracture is slip-induced, cleavage cracks propagate in an unstable manner when the local tensile stress (σ_{yy}) ahead of a stress concentrator exceeds a critical value (σ_f), which is relatively independent of temperature and strain rate (OROWAN, 1948, and KNOTT, 1966).

Values of σ_f have mainly been calculated using rigid-plastic solutions for notched bars loaded in plane strain bending. Here, the longitudinal stress (σ_{yy}) within the plastically deforming zone is derived from slip-line field theory. If fracture loads are well below general yield, plasticity is confined close to the notch root. Hence, if the root is rounded, and if it is assumed that slip-line theory adequately describes the stress state within the plastic region, exponential spiral slip-lines emanate from the root, and σ_{yy} at a distance X below the root is (HILL, 1950)

$$\sigma_{yy} = 2k[1 + \ln(1 + X/\rho)], \qquad X \leqslant r_y, \tag{1}$$

where k is the shear yield stress, ρ is the root radius, and r_y is the plastic zone dimension. This applies up to a maximum value of σ_{yy}, attained when the plastic zone size reaches a critical value r_y' (Fig. 1), and σ_{yy} is then

$$\sigma_{yy}{}^{\max} = 2k[1 + \ln(1 + r_y'/\rho)]. \tag{2}$$

This value can be assumed to remain approximately constant up to the plastic–elastic interface, the precise condition being that slip-lines from the interface pass continuously through yielded material to the straight notch flanks, as illustrated. This maximum value is determined by the notch flank angle θ (GREEN and HUNDY, 1956). At general yield, the value of $\sigma_{yy}{}^{\max}$ at the plastic–elastic interface is

$$\sigma_{yy}{}^{\max} = 2k[1 + \tfrac{1}{2}\pi - \tfrac{1}{2}\theta], \qquad r_y > X > r_y', \tag{3}$$

provided that the included angle θ is greater than 6·4°.

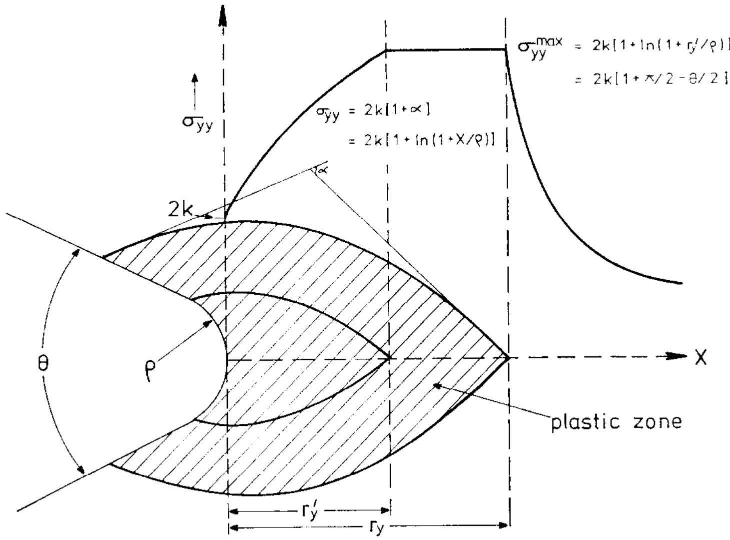

FIG. 1. Schematic longitudinal stress (σ_{yy}) distribution ahead of a rounded notch, radius ρ, at general yield; rigid/plastic slip-line field solution. ($\sigma_y = 2k$, Tresca criterion, and $= \sqrt{3}k$, von Mises criterion.)

Many workers (e.g. KNOTT (1966, 1967) and WILSHAW, RAU and TETELMAN (1968)) have used such methods to verify the constancy of σ_f for the low temperature cleavage of mild steel. It has been found, however, that σ_f remains constant only if cleavage is slip-induced. Where cleavage initiation involves twinning, the value of σ_f becomes markedly sensitive to both temperature and strain rate (OATES, 1968, 1969). More recently, GRIFFITHS and OWEN (1971) have reanalyzed earlier data for cleavage initiation in silicon–iron (GRIFFITHS and OATES, 1969) using a precise plastic–elastic finite element stress analysis for 45° V-notch specimens, of macroscopic root radii, and have found an essentially constant value of σ_f over the slip-nucleated cleavage range from -150 to $+50$°C.

The Orowan σ_f-concept establishes a local criterion for unstable cleavage fracture, which can be related to microstructural features using the model of cleavage cracking derived by SMITH (1966, 1968). Here, cracks are assumed to nucleate from twinning or slip dislocation pile-ups at brittle grain boundary cementite particles, and the critical event is considered to be the growth of these nuclei into the surrounding ferrite matrix (i.e. into a region of higher fracture surface energy). The value of σ_f can be equated to the critical tensile stress required to cause this unstable propagation. Using this model, predictions can be made of the influence of microstructure on σ_f; in particular, the effects of carbide thickness, grain size, temperature and nucleation mechanism can be assessed (OATES, 1968, 1969).

The object of the present study is to relate this local fracture criterion to the macroscopic fracture toughness for a body containing a sharp pre-crack, by examining the temperature dependence of K_{Ic} for the low temperature cleavage failure of mild steel where values of σ_f are known. Here, the significant feature is that the cleavage stress level is exceeded locally at the crack tip even before fracturing, so that one must consider not only the value of σ_f but also the *size scale* over which the criterion is to be met (RICE and JOHNSON, 1970).

2. EXPERIMENTAL METHODS

The steel chosen for the investigation was a high-nitrogen mild steel of Acid Bessemer rimming quality, supplied in the form of 24 mm square hot rolled bars. The composition is shown in Table 1.

TABLE 1

Composition of high-nitrogen mild steel in wt. %

C	Mn	Si	S	P	N
0·07	0·33	0·005	0·053	0·049	0·021

The steel was heat-treated in vacuo for 24 hr at 980°C, followed by a controlled furnace cool of 50°C/hr, to give a uniform equi-axed ferrite grain size of 60 μm. The microstructure of the steel showed evidence of banding of alternate layers of pearlite and ferrite from the previous hot rolling. The carbide was present also as thick grain boundary cementite films, often surrounding the coarse pearlite regions.

Test-pieces were machined as 20 mm square single edge notched (SEN) bars, the geometry of which is shown in Fig. 2. The test-pieces were fatigue pre-cracked, in three-point bend, to an initial crack-length to width ratio (a_0/W) of 0·60, and subsequently heat-treated, ensuring the removal of fatigue damage.

Specimens were deformed in four-point (pure) bending on a 250 kN servo controlled electro-hydraulic Mand testing machine operating under displacement control at 0·4 mm/min, over a range of temperatures from -150 to $+150$°C. The specimens were tested in a constant temperature bath capable of control to ± 1°C. A mixture of Isceon 12 and liquid nitrogen was used for obtaining sub-zero temperatures; tests at temperature above ambient were carried out using heated oil baths.

At low temperatures fracture loads were measured at the point of catastrophic failure (coincident with initiation). At temperatures above -40°C, no such unstable

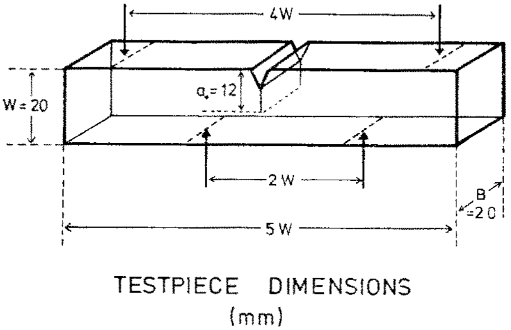

FIG. 2. Geometry of a single edge notched bend test-piece. (Specimens were fatigue pre-cracked to a crack length of 12 mm, i.e. $a_0/W = 0·60$.)

fracture occurred and fracture loads were now measured at initiation. The point of initiation could often be identified from inflections in the load–displacement curves, but at higher temperatures where this was not possible, the electrical potential method was used. The procedure adopted has been discussed elsewhere (RITCHIE, 1972, 1973). The mode of initiation at each temperature was determined by breaking open unfractured test-pieces in liquid nitrogen, and examining the fracture surfaces both optically and in the scanning electron microscope.

Uniaxial tensile properties were determined over the same range of temperatures by testing standard Hounsfield 'No. 13' specimens in a 60 kN single screw-driven Mand testing machine operating at a cross-head speed of 2 mm/min.

3. EXPERIMENTAL RESULTS

The variation in uniaxial tensile properties with temperature for the range -150 to 250°C is shown in Fig. 3. The ductile/brittle transition temperature for the unnotched tensile test-pieces occurred at ≈ -110°C; above this temperature the mode of failure was the standard cup-and-cone type fracture, and below it failure was initiated by cleavage. At temperatures above $+80$°C, serrations during stress–strain curves were observed due to dynamic strain ageing, leading to a ductility trough at $+200$°C ('blue brittleness').

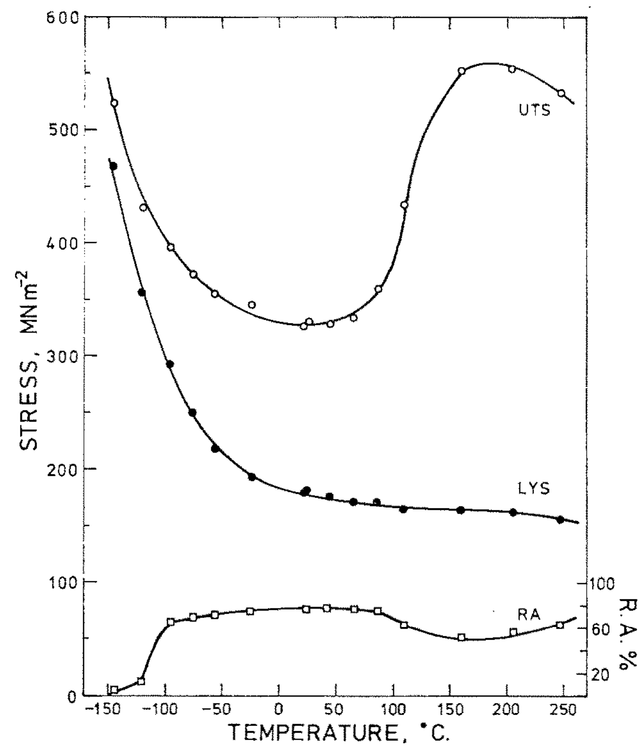

FIG. 3. Variation of uniaxial tensile properties for high-nitrogen mild steel over the temperature range -150 to 250°C. Plotted are the lower yield stress (LYS), the ultimate tensile stress (UTS), and the reduction in area (RA).

The notched bar results are shown in Fig. 4, where stresses have been plotted in terms of the nominal bending stress; the nominal stress below the crack tip (ignoring the stress concentration) is

$$\sigma_{\text{nom}} = 6M/(B(W-a)^2) \text{ for four-point bending,} \tag{4}$$

where B is the specimen thickness, $W-a$ is the depth of uncracked cross-section, and M is the applied bending moment. General yield stresses are also plotted, and can be seen to be in good agreement with those predicted from slip-line field theory (GREEN and HUNDY, 1956), indicating that the deformation approaches that of plane strain.

At temperatures less than -95°C, fracture occurred prior to general yield by slip-induced cleavage leading to immediate catastrophic failure. At temperatures above -95°C, initiation of fracture occurred after general yield and above -60°C involved a non-catastrophic cleavage mode which persisted up to $+90$°C. Above this temperature (the cleavage/fibrous transition temperature T_T), fracture initiated by microvoid coalescence.

The value of the critical tensile stress (σ_f) for cleavage was calculated, using (3), at the temperature (-95°C) where fracture was coincident with general yield. If von Mises' criterion for yielding is assumed, a value of 860 MN m^{-2} can be obtained for σ_f, but this cannot be regarded as *strictly* accurate since it is based on data from sharp-cracked specimens and (3) is not valid for notch angles less than 6·4°. However, the error involved is small, and the value for σ_f obtained compares very closely with a value of 830 MN m^{-2} determined previously for the same steel with a slightly larger grain size of 70 μm (KNOTT, 1967).

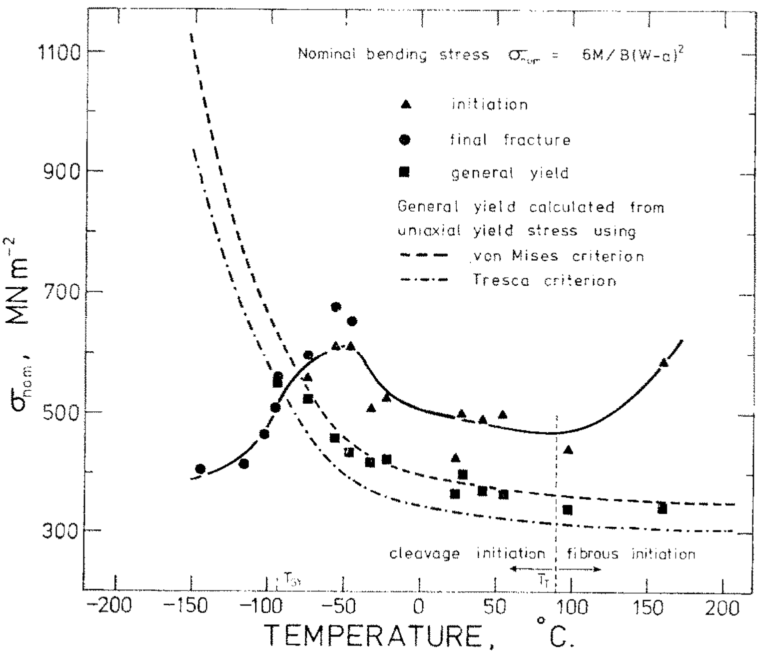

FIG. 4. Experimental general yield and fracture stresses (plotted in terms of nominal bending stress) vs. temperature for pre-cracked bend specimens. Fracture stresses are plotted at initiation and at final failure; these stresses are coincident for temperatures $\lesssim -90$°C.

4. Calculation of Critical Stress Intensity Factors

The standard fracture toughness (K_{Ic}) of the steel was determined for a range of temperatures from -150 to -75°C, where final failure coincided with unstable cleavage propagation. The experimental values of fracture toughness K_Q were determined from the expression derived by Wilson (1970) for pure bending, using

$$K_Q = YM/\{B(W-a)^{3/2}\}, \tag{5}$$

where Y is the calibration factor, taken as 4 for $a/W \geqslant 0{\cdot}6$. The variation of K_Q with temperature is shown in Fig. 5.

Standard fracture toughness testing specifications for linear elastic conditions require that

$$(B, a, W-a) > 2{\cdot}5(K_Q/\sigma_y)^2 \tag{6}$$

be satisfied for valid results, where σ_y is the uniaxial yield stress. This condition was satisfied for values of K_Q at temperatures below -115°C, but above this temperature the size of the plastic zone at fracture was too large, with respect to crack length, to satisfy this validity criterion. Corrections to K_Q were applied to allow for the increasing contribution from plasticity. The plastic zone size, r_y, developed at the crack tip in plane strain can be approximated by

$$r_y = \frac{1}{6\pi}(K/\sigma_y)^2, \tag{7}$$

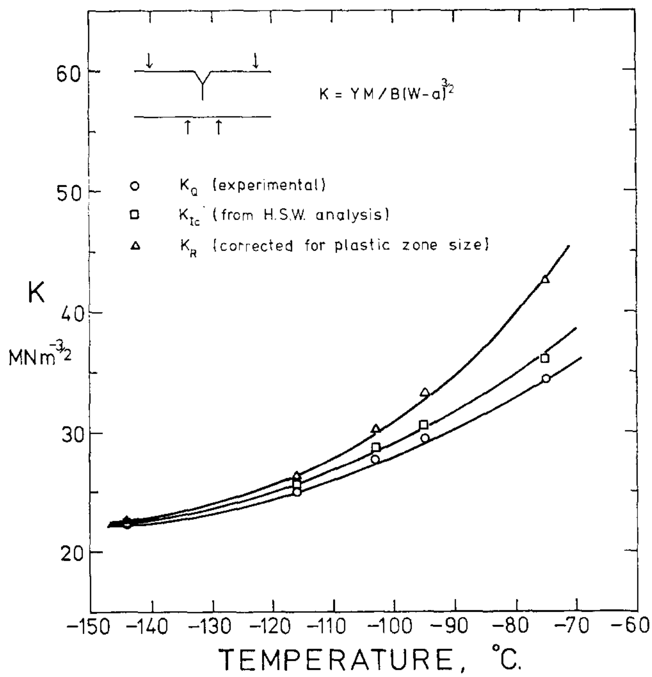

Fig. 5. Variation of critical values of stress intensity at failure. Corrections to experimental values, K_Q, taken using plastic zone size approximations, K_R, and analysis due to Heald, Spink and Worthington (1972) (H.S.W.), K_{Ic}.

and using this measure the value of K_Q can be corrected for plasticity by assuming the effective crack length is increased to $a+r_y$. Since this plastic zone size correction does not provide an accurate description of K when yielding becomes extensive, an alternative correction factor was applied, as used by HEALD, SPINK and WORTHINGTON (1972). This is based on the crack model of BILBY, COTTRELL and SWINDEN (1963) and relates the fracture toughness K_{Ic} to the experimental value K_Q by

$$K_{Ic} = \left[\frac{-8}{\pi} \sigma_u^2 a \ln \{\cos [(K_Q/2\sigma_u)(\pi/a)^{1/2}]\} \right]^{1/2}, \tag{8}$$

where the characteristic flow stress is now taken as the ultimate tensile stress, σ_u. The choice of σ_u is arbitrary, but seems to give good agreement with a number of experimental toughness results.

The variation of the critical values of the stress intensity factor is shown in Fig. 5. It is clear that the plasticity corrections make very little difference to the temperature dependence of fracture toughness over the range studied.

5. MODEL FOR TEMPERATURE DEPENDENCE OF K_{Ic}

It has been established for V-notched bars, of macroscopic root radii, that the plane strain slip-induced cleavage fracture of mild steel is governed by a critical tensile stress criterion, the critical stress σ_f being largely independent of temperature. Considering instead *sharply pre-cracked* specimens, it is therefore postulated that the temperature dependence of K_{Ic} will be determined by the requirement that the maximum principal stress σ_{yy} at the crack tip equals or exceeds this critical value over a microstructurally significant size scale. The recent development of precise analyses for the plastic–elastic stress distribution ahead of sharp cracks makes it possible to test this hypothesis in a quantitative manner. Here, we refer to the asymptotic studies of plane strain crack tip singularities by RICE (1968), RICE and ROSENGREN (1968), and HUTCHINSON (1968), to the subsequent finite element solutions (based on elements which embed the appropriate strain singularity among their admissible deformation fields) by LEVY, MARCAL, OSTERGREN and RICE (1971), RICE and TRACEY (1973), and TRACEY (1973), and to the study of RICE and JOHNSON (1970) on the local alterations of the stress and strain states due to the finite geometry changes involved in progressive blunting of the tip. Typical stress distributions are shown in Fig. 6, where the stress intensification R (ratio of tensile stress σ_{yy} to the flow stress σ_0) is plotted against the dimensionless quantity $X/(K/\sigma_0)^2$, where X is the coordinate of a material point before deformation, and K is the stress intensity factor. Fig. 6a shows Ostergren's finite element solution for a non-hardening material under conditions of small scale yielding. This stress distribution is akin to the conventional elastic distribution with no discontinuity present at the plastic–elastic interface. The positions of the interface directly ahead of the crack and at its maximum extent (inclined at $\approx 70°$ to the line of the crack) are indicated respectively by the first and second carets on the abscissa. The second distribution (Fig. 6b) is the singularity solution for a strain-hardening material ($n = 0$ to $0{\cdot}2$) derived by HUTCHINSON (1968) and RICE and ROSENGREN (1968), again based on the conventional small geometry change (SGC) assumptions. The SGC solutions predict infinite stresses and no intense strain concentration directly ahead of the tip, and consequently an approximate modified stress distribution is shown near the singularity to allow for crack tip blunting (RICE and JOHNSON, 1970).

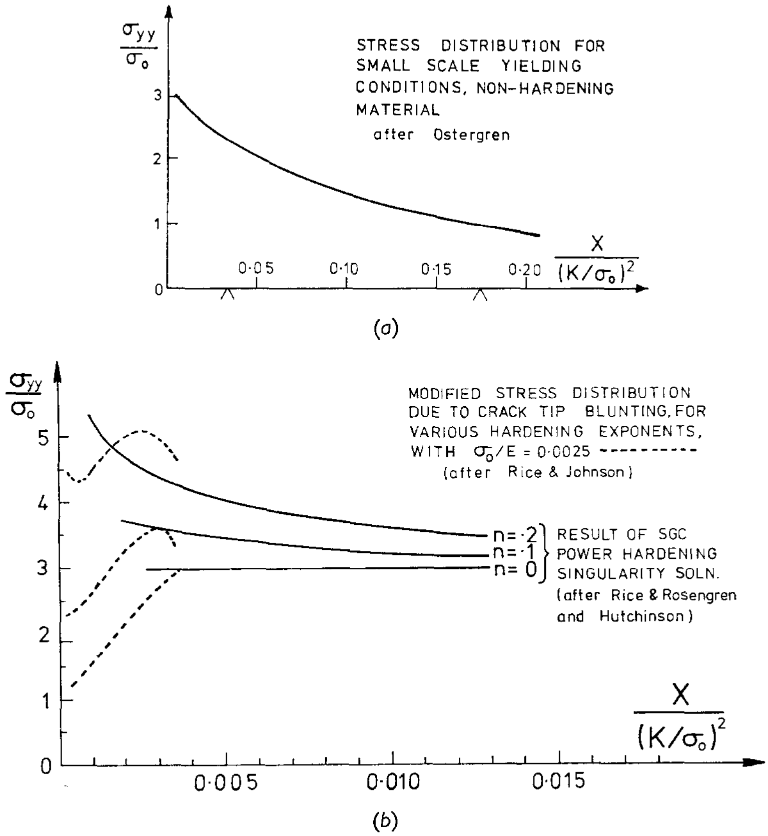

FIG. 6. Distribution of longitudinal stress (σ_{yy}) acting directly ahead of a sharp crack in plane strain for (a) small scale yielding (SGC) conditions for non-hardening material from finite-element computer solution due to OSTERGREN, and (b) small scale yielding conditions from singularity solution for hardening material due to RICE and ROSENGREN (1968) and HUTCHINSON (1968) (solid lines). Near tip stress distribution in (b) modified for initial yield strain σ_0/E of 0·0025 due to RICE and JOHNSON (1970) (non-solid lines). Carets in (a) indicate the positions of the plastic–elastic interface directly ahead of the crack and the maximum extent of the plastic zone at $\approx 70°$ to the line of the crack. (After RICE and JOHNSON (1970).)

If the flow stress σ_0 is taken to be the uniaxial yield stress, these solutions enable the value of the longitudinal stress (σ_{yy}) to be calculated at any distance ahead of the crack tip. We assume that fracture occurs when σ_{yy} reaches a critical value. At low temperatures little stress intensification is needed since σ_0 is high and the critical fracture stress can be met by a point on the Ostergren curve (Fig. 6a) not too far behind the plastic–elastic interface. The absolute size of the plastic zone for a given value of K is small because the yield stress is high. As the temperature is raised, increasing stress intensification is required, which now involves work-hardening as well as constraint, because the strains in the plastic zone become significant. Hence, the failure point is now found on the power hardening curve (Fig. 6b) and moves progressively closer to the maximum in the curve as the temperature is raised.

It is clear from these near tip stress distributions that the maximum stress intensification possible can be very much larger than those predicted for rounded notches by slip-line field theory (Fig. 1), and that it occurs not at the plastic–elastic interface, but very much closer to the crack tip.

If the fracture criterion in a sharp-cracked specimen were simply that σ_{yy} should be sufficiently large to exceed a critical value, σ_f, it is apparent that fracture could be produced, very close to the crack tip, by vanishingly small applied loads. Hence, it seems necessary to supplement such a criterion by the additional requirement that the critical stress be achieved over some microstructurally significant distance (the 'characteristic' distance) ahead of the tip. We suppose that the critical stress at any temperature is attained at some *fixed* distance ahead of the tip, rather than at the plastic–elastic interface. Cleavage fracture is generally associated with the cracking of grain boundary carbides, and at first sight it is feasible to suppose that the characteristic distance should be of the order of one grain diameter; i.e. the distance of the first grain boundary carbide from the crack tip. SMITH's (1966, 1968) model, however, which has been used successfully to relate the value of σ_f to microstructure in notched bars, assumes that a microcrack formed within such a carbide grows into the ferrite matrix under the action of a uniform tensile stress. In the plastic zone ahead of the macroscopic crack tip the stress is non-uniform, and it is plausible that insufficient tensile stress is generated across the second grain to propagate the carbide crack nucleus if the critical stress predicted by Smith's theory is achieved only at the first boundary. This would suggest a characteristic distance of something greater than one diameter (see also TETELMAN, WILSHAW and RAU (1968)).

We postulate that for cleavage failure the magnitude of K_{Ic} is governed by the size of the plastic zone when σ_{yy} exceeds σ_f over the characteristic distance. It is clearly possible then to predict values of K_{Ic} using the stress distributions directly from a knowledge of the flow stress and σ_f. Then, taking the value of σ_f to be 860 MN m^{-2} over the temperature range in question, and the flow stress σ_0 as the uniaxial yield stress at the required temperature (Fig. 3), a measure of plastic zone size (in terms of $(K/\sigma_0)^2$), and hence the fracture toughness K_{Ic}, can be calculated as a function of temperature. For the power hardening solution a value for the work-hardening coefficient n was taken as 0·1, which represented the average value measured in the low-temperature tensile tests. The results of these calculations for the cleavage fracture of mild steel between -150 and -75°C are shown in Fig. 7. The predicted values of toughness represent the value of K at which a stress of 860 MN m^{-2} is achieved at distances ahead of the crack tip of 60 and 120 μm respectively. It can be clearly seen that, for a characteristic distance of two grain diameters, agreement is very close to the experimentally determined variation of K with temperature. In fact, TRACEY's (1973) recent numerical solutions for power-law hardening materials reveal the actual continuous transition from near tip stress distributions of the kind in Fig. 6(b) to far distributions similar to that of Fig. 6(a). This has the effect of smoothing the predicted K-values in Fig. 7.

Attempts to extend this analysis to characterize fracture at higher temperatures proved unsuccessful. This could be due to one of several reasons. Firstly, the stress distribution is derived in terms of the linear elastic stress intensity factor K. As the testing temperature increased, the plasticity contribution became extremely large, and the value of K in specimens of small dimensions is much less meaningful. Further, for temperatures greater than approximately -60°C, the mode of the cleavage failure changed. At temperatures below -60°C, catastrophic final failure occurred immediately on initiation of a microcrack. Between −75 and −45°C, a region of 'stable' cleavage growth occurred before final failure. At higher temperatures, no catastrophic failure could be identified, and the entire growth involved the plastic growth and coalescence of cleavage microcracks. It has been suggested (TETELMAN and WILSHAW, 1969) that such cleavage propagation is strain controlled, and hence the present analysis based on a critical stress criterion would not be applicable.

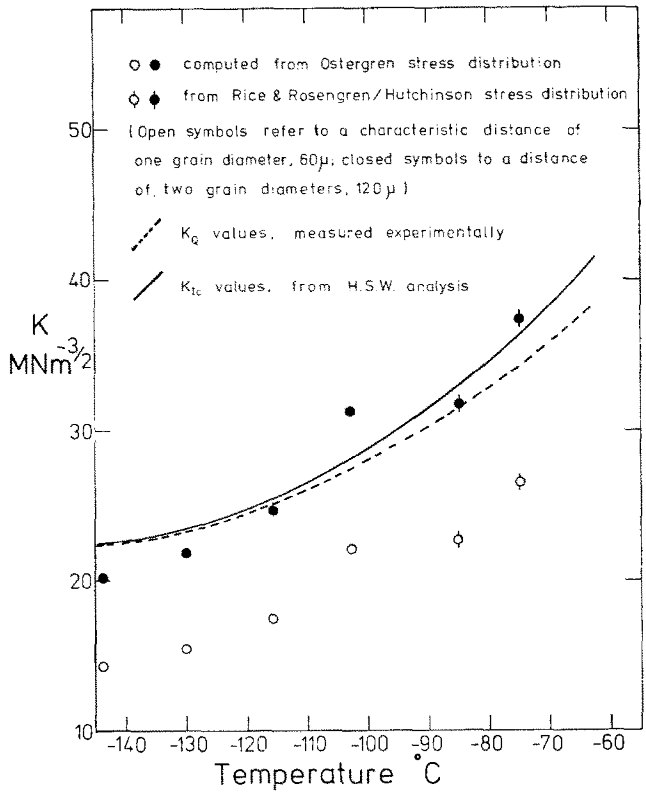

FIG. 7. Comparison of the variation of fracture toughness with temperature between experimental values (K_Q and K_{Ic}) and predicted values for characteristic distances of one and two grain diameters.

6. DISCUSSION

A previous model relating the σ_f local fracture criterion to K_{Ic} (WILSHAW, RAU and TETELMAN, 1968, and TETELMAN, WULLAERT and IRELAND, 1971) was similarly based on the attainment of a critical value of σ_{yy} equal to σ_f ahead of a stress concentrator. The analysis was formulated for fracture in round-notched specimens where slip-line field theory predicts the maximum stress intensifications to occur at the plastic–elastic interface at a plastic zone size r_y' given by (2). By rearranging this equation, the critical plastic zone size r_c was obtained where the σ_{yy} stress had reached σ_f, for a notch of root radius, ρ, viz.

$$r_c = \rho[\exp(\sigma_f/\sigma_y - 1) - 1], \qquad r_c < r_y', \tag{9}$$

assuming Tresca's yield criterion.

Further, for a sharp crack, as is often assumed in linear elastic fracture mechanics studies,

$$r_c \approx 0{\cdot}12(K_{Ic}/\sigma_y)^2. \tag{10}$$

By equating these two relations WILSHAW, RAU and TETELMAN (1968) obtained the expression for a notch of root radius ρ:

$$K_{Ic}(\rho) = 2{\cdot}9\sigma_y[\exp(\sigma_f/\sigma_y - 1) - 1]^{1/2}\rho^{1/2}. \tag{11}$$

The 'true' fracture toughness K_{Ic}, i.e. that relating to fracture ahead of a sharp (fatigue) crack, was then obtained by replacing ρ by a limiting value, ρ_0, determined empirically. It is important to realise that the analysis is based on a stress distribution ahead of a *round* notch where the failure point (i.e. where σ_{yy} exceeds the critical value of σ_f) is at the plastic–elastic interface. This demands that the plastic zone size at failure, r_c, must be smaller than r_y' so that the deformation can be described in terms of Hill's logarithmic spirals (see equation (2)). Thus, over a range of temperature, (with corresponding variation in yield stress), the failure point ahead of a round notch will occur at *varying* distances from the notch tip, simply given by $X = r_c < r_y'$, because the size of this critical zone will be governed by the yield stress. This, however, does not describe the situation ahead of a sharp crack, where the failure point is situated at a *fixed* distance ahead of the crack tip, and σ_{yy} must exceed σ_f not merely at a point but over the characteristic distance.

This apparent contradiction between fracture ahead of a rounded notch and ahead of a sharp crack can be resolved by considering the different forms of stress distribution ahead of each stress concentrator (Figs. 1 and 6). It is clear that the possible intensification of tensile stress is larger ahead of a sharp crack, and the position of the maximum intensification is very much closer to the notch tip. In fact if we examine the failure situation for the present results at -144 and -75°C (the extremes of the range studied), we can see from Fig. 6 that (i) at -144°C, $R_{\max}$ is obtained at 6 μm from the crack tip when the plastic zone size is 120 μm and (ii) at -75°C, $R_{\max}$ is at 60 μm when the plastic–elastic interface is 1000 μm from the crack tip. Thus, at all temperatures studied the position of $R_{\max}$ at failure will occur at or before the first grain boundary, and hence the stress gradient ahead of this point will be decreasing.

Now, in mild steel unstable cleavage fracture can result only if the tensile stress sufficient to initiate a crack at a grain boundary carbide is also sufficient to propagate it through the next grain boundary, i.e. σ_{yy} must exceed σ_f over at least one grain diameter (d). Ahead of a sharp crack the necessary stress intensification occurs very close to the tip and hence initiation ($\sigma_{yy} > \sigma_f$ at a point) can occur at the first grain boundary carbide, that is at about one grain diameter from the tip. For unstable fracture, σ_{yy} must exceed σ_f over the next grain, and since σ_{yy} will be decreasing with increasing distance, the load must be raised such that $\sigma_{yy} > \sigma_f$ at two grain diameters from the tip. Thus, we arrive at a condition that σ_{yy} must exceed σ_f at a characteristic distance of approximately two grain diameters from the crack tip.

Ahead of a rounded notch, however, the necessary stress intensification for a similar value of K will be reached at a distance much further from the notch tip; in fact, at the plastic–elastic interface ($X = r_c > 2d$). The characteristic distance can thus be regarded as the limiting value of X necessary for unstable cleavage fracture. TETELMAN, WILSHAW and RAU (1968) took this value of X to mean the limiting value of

$r_y(=r_c)$, and thus for a notch sharper than the limiting root radius ρ_0 the criterion of $\sigma_{yy} > \sigma_f$ over one grain diameter would be reached for a value of r_c less than the characteristic distance. Although correct in essence, this does not describe the full situation since for the sharp notch the failure point will not be at the plastic–elastic interface.

Thus, summarizing, unstable cleavage fracture will occur at a distance X from the notch tip where σ_{yy} exceeds σ_f over at least one grain diameter. For a rounded notch (root radius $> \rho_0$) this will occur at the plastic–elastic interface at a value of X equal to r_c (assuming the logarithmic spiral slip-line field within the plastic region). For a sharp crack, the maximum stress intensification is achieved at a fraction of the plastic zone size, and so the failure criterion can be achieved within the characteristic distance, which will represent the smallest value of X necessary for unstable fracture. From microstructural considerations, we have shown that this limiting distance is about two grain diameters ($X = 2d$) for an unstable cleavage fracture in a coarse grained mild steel. This situation, for a sharp crack, is shown schematically in Fig. 8.

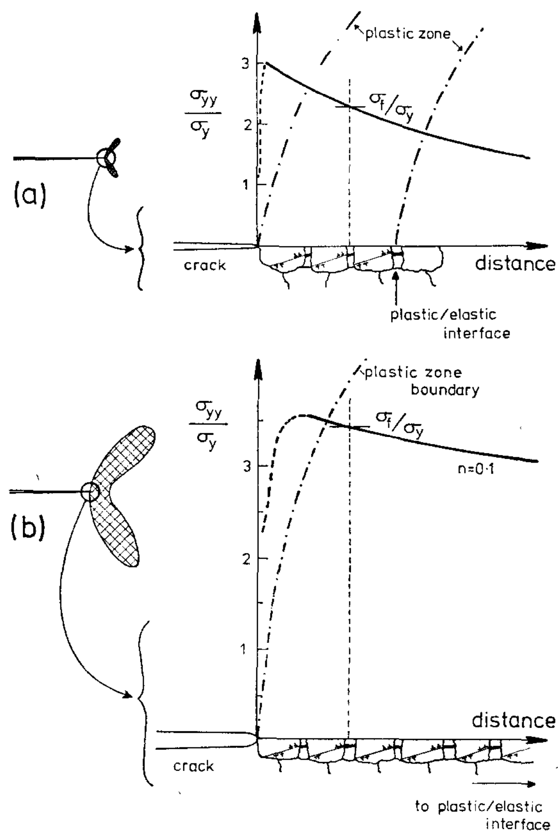

FIG. 8. Schematic representation of the critical fracture event. (a) Situation at low temperature. The critical fracture stress is attained at the second grain boundary ahead of the crack. Because the yield stress is high, the stress intensification need not be high and the critical event can occur close to the plastic–elastic interface. The plastic zone and hence K_{Ic} can therefore be small. (b) Situation at higher temperature. A high stress intensification is now needed, because the yield stress is lower. The plastic zone size at fracture and hence K_{Ic} must be larger.

It is interesting to note that the slip-line field solution for rounded notches predicts initiation at the plastic–elastic interface. Recent work by GRIFFITHS and OATES (1969), however, has shown that initiation ahead of such notches may well occur behind the interface. At first sight this suggests that a certain plastic strain is necessary to achieve a significant number of cracked carbides for initiation, and therefore appears to lend strong support to the fibre-loading model for cleavage crack initiation (LINDLEY, OATES and RICHARDS, 1970). However, the more recent finite element analysis for the stresses in a 45° V-notched bar in bending (GRIFFITHS and OWEN, 1971, and OWEN, NAYAK, KFOURI and GRIFFITHS, 1973) has shown that the position of the maximum stress intensification is in fact some distance behind the plastic–elastic interface (Fig. 9). This is then similar to the situation ahead of a sharp crack (Fig. 6) where the maximum longitudinal stress can occur well before this interface close to the crack tip.

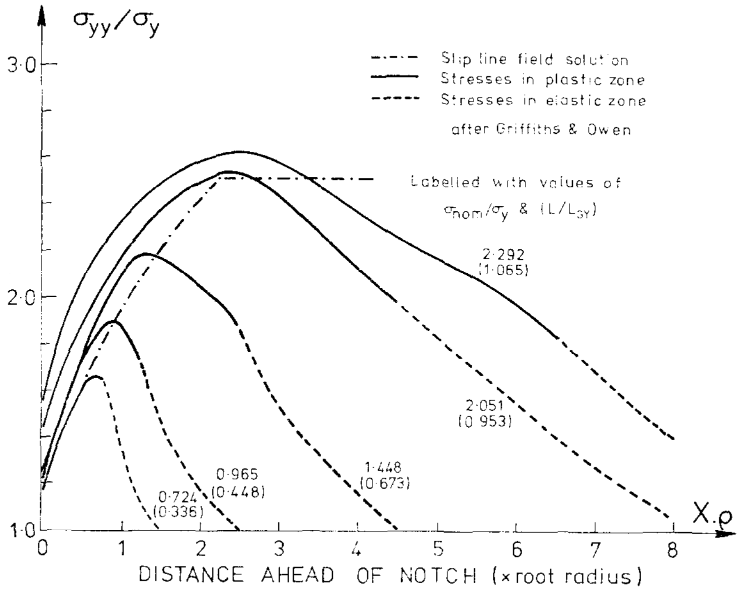

FIG. 9. Distribution of longitudinal stress (σ_{yy}) acting directly ahead of a rounded notch ($\theta = 45°$) in plane strain at various loads (L) from plastic–elastic finite-element solution due to GRIFFITHS and OWEN (1971). For material obeying von Mises' yield criterion with linear work-hardening.

The Tetelman–Wilshaw model is based on the slip-line field solution for rounded notches, and therefore predicts initiation at the interface ($X = r_c$). In fact the correlation between σ_f and K_{Ic} is based on the size of the plastic zone, r_c, at this point (equation (10)), although (10) will not strictly be valid for rounded notches. The present procedure removes this difficulty of the precise relationship between the failure point and the plastic zone size since it is possible to calculate K_{Ic} directly from a knowledge of the characteristic distance, the yield stress and the σ_f-value from the stress distribution in Fig. 6.

One final point is that the magnitude of the maximum possible stress intensification R_{max} can give some indication of the temperature of the cleavage/fibrous transition. If the maximum achievable stress is insufficient to equal the critical value σ_f, fracture cannot initiate as cleavage (RICE and JOHNSON, 1970). The fracture mode at this point must then change to one of fibrous rupture. The magnitude of this maximum achievable stress of course will be very much dependent on the yield stress and hardening properties. For the present results, the work-hardening coefficient can be taken as 0·2, and the maximum stress intensification from Fig. 6(b) as 5. The temperature of the cleavage/fibrous transition (T_T) was found to be +90°C, and at this temperature the yield stress is 168 MN m^{-2}. Thus, if the onset of fibrous rupture at +90°C coincides with the condition that $\sigma_{yy}^{\max} < \sigma_f$, and if $R_{\max} = \sigma_{yy}^{\max}/\sigma_y = 5$ and $\sigma_y = 168$ MN m^{-2}, then $\sigma_f \geqslant R_{\max}\sigma_y = 840$ MN m^{-2}. This is extremely close to the experimental value of σ_f equal to 860 MN m^{-2} (at −95°C). In theory, the method can be used in reverse to predict the temperature of the transition, but this is difficult in the present case because the variation in yield stress with temperature over this range is very small.

Acknowledgments

The writers would like to thank Professor R. W. K. Honeycombe for provision of laboratory facilities and the Director of Computing Services, Cambridge University for use of the TITAN computer. Financial support from the Science Research Council for one of the writers (R. O. R.) is gratefully acknowledged, whereas another writer (J. R. R.) wishes to acknowledge support of the United States Atomic Energy Commission and of an Overseas Fellowship at Churchill College, Cambridge.

References

Bilby, B. A., Cottrell, A. H. and Swinden, K. H. 1963 *Proc. R. Soc.* **A272**, 304.

Green, A. P. and Hundy, B. B. 1956 *J. Mech. Phys. Solids* **4**, 128.

Griffiths, J. R. and Oates, G. 1969 *Fracture 1969. Proceedings of the Second International Conference on Fracture* (edited by Pratt, P. L.), p. 229. Chapman and Hall, London.

Griffiths, J. R. and Owen, D. R. J. 1971 *J. Mech. Phys. Solids* **19**, 419.

Heald, P. T., Spink, G. M. and Worthington, P. J. 1972 *Mater. Sci. Eng.* **10**, 129.

Hill, R. 1950 *The Mathematical Theory of Plasticity*. Oxford University Press.

Hutchinson, J. W. 1968 *J. Mech. Phys. Solids* **16**, 13.

Knott, J. F. 1966 *J. Iron Steel Inst.* **204**, 104.

1967 *J. Mech. Phys. Solids* **15**, 97.

Levy, N., Marcal, P. V., Ostergren, W. J. and Rice, J. R. 1971 *Int. J. Fracture Mech.* **7**, 143.

Lindley, T. C., Oates, G. and Richards, C. E. 1970 *Acta Met.* **18**, 1127.

Oates, G. 1968 *J. Iron Steel Inst.* **206**, 930.

1969 *Ibid.* **207**, 353.

Orowan, E. 1948 *Rep. Prog. Phys.* **12**, 185.

Owen, D. R. J., Nayak, G. C., Kfouri, A. P. and Griffiths, J. R. 1973 *Int. J. Num. Meth. Engng.* **6**, 63.

Rice, J. R. 1968 *J. Appl. Mech.* **35**, 379.

Rice, J. R. and Johnson, M. A. 1970 *Inelastic Behavior of Solids* (edited by Kanninen, M. F., Adler, W., Rosenfield, A. and Jaffe, R.), p. 641. McGraw-Hill, New York.

RICE, J. R. and ROSENGREN, G. F. 1968 *J. Mech. Phys. Solids* **16**, 1.

RICE, J. R. and TRACEY, D. M. 1973 *Numerical and Computational Methods in Structural Mechanics* (edited by FENVES, S. J., PERRONE, N., ROBINSON, A. R. and SCHNOBRICH, W. C.). Academic Press, New York. In press.

RITCHIE, R. O. 1972 Dep. of Metallurgy and Materials Science, Cambridge University, Internal Report.

1973 Ph.D. Thesis, Cambridge University.

SMITH, E. 1966 *Physical Basis of Yield and Fracture:* Conference Proceedings, p. 36, Institute of Physics and Physical Society, London, p.36.

1968 *Int. J. Fracture Mech.* **4**, 131.

TETELMAN, A. S. and WILSHAW, T. R. 1969 *Fracture 1969. Proceedings of the Second International Conference on Fracture* (edited by PRATT, P. L.), p. 219. Chapman and Hall, London.

TETELMAN, A. S., WILSHAW, T. R. and RAU, C. A. 1968 *Int. J. Fracture Mech.* **4**, 147.

TETELMAN, A. S., WULLAERT, R. A. and IRELAND, D. 1971 *Practical Applications of Fracture Mechanics to Pressure-Vessel Technology*, p. 85. Institution of Mechanical Engineers, London.

TRACEY, D. M. 1973 Ph.D. Thesis, Brown University.

WILSHAW, T. R., RAU, C. A. and TETELMAN, A. S. 1968 *Eng. Fract. Mech.* **1**, 191.

WILSON, W. K. 1970 *Ibid.* **2**, 169.