E(5) Critical-Point Behavior


Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720

Notable benchmarks of collective nuclear behavior are the harmonic vibrator, the symmetrically deformed rotor, and the triaxially soft rotor. They correspond to limits of the Interacting Boson Model (IBM). While nuclei may display behavior near to these idealized limits, many lie in transitional regions between them. Algebraic descriptions of the nature of the transition have been developed in direct analogy with classical phase transitions. Recently, it has been suggested that a useful approach is to apply the idea of a critical point of the shape change as a new benchmark against which nuclear properties can be compared. In particular, the transition from a spherical harmonic vibrator to an axially deformed rotor has been described analytically [1] by introducing a dynamic symmetry, denoted as X(5), which arises when the potential in the Bohr Hamiltonian is decoupled into two components – an infinite square well potential for the quadrupole deformation parameter, β, and a harmonic potential well for the triaxiality parameter, γ. This is an approximation of the “true” potential found at the critical point of the shape change from the IBM calculations. Several examples of nuclei that may be close to the X(5) critical point have been suggested and we recently published the results of an extensive search for candidate X(5) nuclei [2].

In this contribution we report on a search [3] to find examples of nuclei near the critical point of the transition from a triaxially soft rotor to a spherically harmonic vibrator, denoted as E(5). This description involves the solution of the Bohr Hamiltonian with an infinite square well potential for the quadrupole deformation parameter, denoted as E(5). This description involves the solution of the Bohr Hamiltonian with an infinite square well potential depending only on β [4]. IBM calculations indicate that this is a fair approximation to the flat-bottomed potential that is calculated at the critical point of the shape transition from gamma-soft rotor to harmonic vibrator. IBM calculations have also been used to account for finite boson number effects which alter the predictions of absolute values, and ratios, of both state energies and transition strengths. The transition strengths are also affected by the level of approximation regarding the quadrupole transition operator. It is therefore of great importance to identify a set of robust characteristics of E(5) behavior that do not change drastically with the details of a given calculation.

Some of the key properties of the E(5) description (see Fig. 1) that we used in our search can be summarized as follows:

1) The energy ratio $E(4^+_1)/E(2^+_1)$ should be $\approx 2.20$.

2) The $B(E2; 4^+_1 \rightarrow 2^+_1)$ value should be $\approx 1.5$ times the $B(E2; 2^+_1 \rightarrow 0^+_1)$ value.

3) There should be two excited $0^+$ (usually labeled $0^+_\gamma$ and $0^+_\beta$) states lying at approximately $3-4$ times the energy of the $2^+_1$ state.

4) The decay of the $0^+_\gamma$ level reflects a multiphonon structure. There is an allowed E2 transition to the $2^+_1$ level, but the transition to the $2^+_1$ level is forbidden.

5) The decay of the $0^+_\gamma$ state should also be characteristic of E(5) behavior. There is an allowed transition to the $2^+_1$ level with a strength $= 0.5$ times the $B(E2; 2^+_1 \rightarrow 0^+_1)$ value.

The first two points reflect the fact that the E(5) behavior lies intermediate between that for a harmonic vibrator and a gamma-soft rotor. While the precise energies of both the excited $0^+$ states are quite sensitive to the details of the calculations their decays are reflective of the underlying E(5) symmetry properties.

If the E(5) description is to be taken as a benchmark for the nuclear shape transitions, then it is important to find examples close to the predicted behavior. We used the key properties described above to search the ENDF data file for examples of even-even nuclei, with $30 \leq Z \leq 82$, $A \geq 60$, which display possible E(5) behavior. Of the $\approx 70$ nuclei with $E(4^+_1)/E(2^+_1)$ values that might indicate such behavior, only six (namely, $^{106}$Pd, $^{106,108}$Cd, $^{126}$Te, $^{128}$Xe, and $^{132}$Ba) have firmly assigned first and second excited $0^+$ states in the range of excitation energy that might be expected for the $0^+_\gamma$ and $0^+_\beta$ excitations of the E(5) picture. The cases of $^{106}$Pd and $^{132}$Ba had already been discussed as possible example of E(5) critical-point nuclei [5,6]. Of the remaining candidates the best agreement between experiment and the E(5) predictions is found to be with $^{128}$Xe (see Fig. 1), which is proposed as a new example of an E(5) nucleus.

Fig. 1: Level scheme calculated for the E(5) symmetry (left) and the empirical scheme for $^{128}$Xe (right). The excitation energies are normalized to the energy of the $2^+_1$ level and the numbers indicate normalized E2 strengths.

REFERENCES