Scale Dependence of Biogeochemical Reaction Rates: Experimental and Modeling Approaches

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Discrepancy between laboratory and field rates may be 3 to 5 orders of magnitude.

Possible explanations:
- Passivation of mineral surfaces with time (White and Brantley, 2003; Maher et al., 2004)
- Slowing of rates close to equilibrium (the “affinity effect”)
- Physical heterogeneity (Malmstrom et al., 2000)
- Geochemical heterogeneity (Li et al., 2006)
- Microbiological heterogeneity (???????)

Might some part of the discrepancy be due to the scale dependence of rates in the subsurface?
- If so, at what scales (pore to pore network to meter) does this scale dependence arise?
- Some studies without physical/chemical heterogeneity have shown a discrepancy (Maher et al, 2006), so this cannot explain 100%
Role of Physical Heterogeneity

Flow Direction

Non-reactive tracer

pH
Role of Geochemical Heterogeneity

Heterogeneous distribution of plagioclase at the pore network scale (Li et al., 2007)

Distribution of reaction rates within pore network
Does A Scale Dependence Occur at the Pore Scale?

Conceptual model for cylindrical pore

Reaction rate that is measured depends on the scale of the measurement
Scaling in the Context of “Realistic” Rate Laws

Calcite Dissolution
One of the most important and fastest mineral reactions in the subsurface.

\[ \text{CaCO}_3(s) + \text{H}^+ = \text{Ca}^{2+} + \text{HCO}_3^- \]

\[ R = Aa_{H^+} \left[ 1 - \left( \frac{Q}{K_{eq}} \right) \right] \]

The reaction rate depends on pH under acidic conditions.

Plagioclase Dissolution
An example of slow silicate dissolution.

\[ \text{Ca}_{0.7}\text{Na}_{0.3}\text{Al}_{1.7}\text{Si}_{2.3}\text{O}_8 + 8 \text{H}_2\text{O} = 0.7 \text{Ca}^{2+} + 0.3 \text{Na}^+ + 1.7 \text{Al(OH)}_4^- + 2.3 \text{SiO}_2(\text{aq}) + 4.6 \text{H}_2\text{O} \]

Hellmann Rate Law
(Hellmann and Tisserand, 2006)

The reaction rate depends on pH and reaction affinity.

\[ R = Aa_{H^+}^{0.7} \left[ 1 - \exp\left( n \Delta G^{m1} \right) \right]^{m^2} \]

Aluminum inhibition (Oelkers) rate law

\[ R = k \left( \frac{K_f}{a_{Al(\text{OH})_3} + K_f} \right) \left[ 1 - \left( \frac{Q}{K_{eq}} \right) \right]^{1/3} \]

The reaction rate depends on pH and dissolved aluminum.

Dissimilatory Iron Reduction
One of the most important biogeochemical reactions.

\[ 4\text{Fe(OH)}_3(s) + \text{CH}_3\text{CHOHCOO}^- + 7\text{H}^+ = 4\text{Fe}^{2+} + \text{CH}_3\text{COO}^- + \text{HCO}_3^- + 10\text{H}_2\text{O} \]

Monod rate law

\[ R = \mu_{\text{max}} B \left( \frac{C_{\text{Fe(OH)}_3}}{K_{Fe(OH)_3} + C_{\text{Fe(OH)}_3}} \right) \left( \frac{C_{\text{Lactate}}}{K_{\text{Lactate}} + C_{\text{Lactate}}} \right) \]
Experimental Validation of Reactive Pore Model

A

Calcite sample (4000 micron thickness)

Supporting ceramics (100 micron thickness)

500 micron diameter hole

B

Ca^{2+} Concentration (µMolar)

Flow rate (µLiters/min)

- Experiment, pH = 5
- Modeling, pH = 5
- Experiment, pH = 4
- Modeling, pH = 4
Results for a Single Calcite Pore

Compare rates from 2D reactive Poiseuille flow ($R_d$) with well-mixed reactor ($R_m$) models

1. **Transport control**
   Pore fluid reaches equilibrium

2. **Mixed control**
   Comparable rates of flow and reaction

3. **Surface reaction control**
   Rates too slow relative to flow
Scaling Effects as a Function of Aperture

Larger aperture increases the diffusion distance, lessening the efficiency of mixing.
Results for a Single Planar Fracture

Geometry controlled by Peclet number, $\text{Pe} = \frac{v \delta}{D}$, but magnitude of scaling effect depends on absolute rate.
Scaling Effect as a Function of Fracture Length

Plagioclase (Al inhibition)

- Transverse Mixing
- Longitudinal Mixing
- Complete Mixing

Calcite

- Transverse Mixing
- Longitudinal Mixing
- Complete Mixing
Capillary Tube Experiments:
Reductive Dissolution of Fe-Hydroxide

- Effluent for ICP-OES and colorimetric analysis
- Inline Filter
- Silica capillary tubing (100~250 um ID, 10~30um wall thickness)
- Detector
- Nano Fe oxides coated silica microspherical particles (20~180um diameter)
- AH2DS & inorganic reductive solution

Ferrihydrite-coated glass spheres

Capillary tube with coated spheres
Reaction Network for Fe-Hydroxide Reduction

- Fe reduction will compete with enzymatic reduction of contaminants (U, Tc, Cr)
- Biogenic Fe can reduce some radionuclides (Tc, U?)

\[ \text{Ferrhydrite Dissolution} \]
\[ \text{Rate} = -\Delta_{FH} \, k \left( > \text{FeOH} \cdot \text{Fe}^+ \right) \left( 1 - \frac{Q}{K_{eq}} \right) \]
> \text{FeOH} \cdot \text{Fe}^+ = \text{strong sites on Fe(OH)}_3 \text{ surface}

\[ \text{Green Rust} \]
\[ \text{Rate} = -\Delta_{GR} \, k \left( 1 - \frac{Q}{K_{eq}} \right) \]
\[ \Delta_{GR} = \{ \text{Precipitation = Arbitrary non-zero value (no nucleation threshold)} \} \]
\[ \{ \text{Dissolution = Specific surface area} \} \]

\[ \text{Goethite} \]
\[ \text{Rate} = \begin{cases} 
\text{Precipitation:} & -\left( A_{Goethite} + \gamma A_{FH} \right) k \left( 1 - \frac{Q}{K_{eq}} \right) \\
\text{Dissolution:} & -A_{Goethite} k \left( > \text{FeOH} \cdot \text{H}^+ \right) \left( 1 - \frac{Q}{K_{eq}} \right) 
\end{cases} \]

\[ \text{Magnetite} \]
\[ \text{Rate} = -\left( A_{Mt} + A_{GR} \right) k \left( 1 - \frac{Q}{K_{eq}} \right) \]

\( A_{Mt} \) is initially 0, Green Rust required as a precursor
Conversion of Ferrihydrite to Magnetite

- Injection of 20 mM FeSO₄
- Monitoring with XRD at Beamline 11.3.1 (ALS) using a 100 µm beam size
- Rate is about $1.3 \times 10^{-10}$ mol/m²/s (about 1 order of magnitude faster than reported by Hansel, Benner, and Fendorf, 2003)
Microporosity and Diffusion Rates:
Example of Weathering Rinds in Basalt

In collaboration with Alexis Navarre-Sitchler and Susan Brantley, Penn State University
μXRF Mapping of Bromide Diffusion Front

Porosity (%) vs. D/D0

- Unweathered Samples (DP3, DP6, DP5)
- DP5 Weathered (Advanced Front)

Concentration vs. Distance (mm)

- 17 Days
- 34 Days
X-Ray Microtomography of Rind Interface

X-ray beam from ALS (Beamline 8.3.2)

4.4 μm 3-D resolution
Connectivity of Porosity

Connected porosity fraction calculated with “burning algorithm” in the code Percolate

Data from microtomography at ALS on basalt

Connectivity versus porosity in cement (Bentz, 2004)
3D Microtomography of Weathered Basalt

Skeletonized version of 125x125x125 pixel microtomographic data
- Red: Pores
- Blue: Matrix
Diffusion of a Bromide Tracer in Rind Interface

Bromide tracer released at bottom boundary, with pixel by pixel effective diffusivity based on microtomography
- Initial time: 0.001 day
- Final time: 0.2 days
Comparison of Model and Experimental Diffusion

Scaled result to 7 days assuming $t^{1/2}$ dependence

μXRF result using bromide tracer
Porosity-Effective Diffusivity Relationship

Archie’s Law gives continuous increase in diffusivity with increase in porosity, even at low values

\[ \frac{D}{D_0} = \phi^2 \]

In Threshold Model, no increase in diffusivity at low porosities due to lack of connectivity

\[ \frac{D}{D_0} = 2 \times 10^{-5} + 0.2(\phi - \phi_{\text{crit}})^2 \]
Investigate effect of higher NaNO₃ concentrations on Sr exchange in sediments contaminated by tank leaks

In the case of Sr (unlike Cs) the divalent cations (Ca, Mg) play a much more significant role in determining sorption, and thus retardation

Dissolution and/or precipitation of calcite limits concentration range of Ca (the most important competing cation), and complicates interpretation of data

Can a model be developed to capture the ionic strength dependence of the selectivity coefficients (especially Sr:Na)?
Experimental Approach

- Systematic batch experiments targeting pseudo-binary Ca:Sr, Ca:Mg, Na:Sr, Na:Ca (experiments by Zachara et al)
  - Carried out at nearly constant normalities of 0.001N, 0.01N, and 0.1N
  - All relevant cations measured in pore solution and on the exchanger using ammonium chloride flush (i.e., no assumption of a binary system)

- Column experiments using 10 µM Sr
  - 10mM NaNO$_3$, 1mM Ca, 1mM Mg
  - 100mM NaNO$_3$, 1mM Ca, 1mM Mg
  - 100mM NaNO$_3$, 0.5mM Ca, 0.01mM Mg
  - 30mM NaNO$_3$, 0.5mM Ca, 0.01mM Mg
Modeling Approach

- Compare and contrast single-site and multi-site models
- Evaluate the possibility that exchange of monovalent ion pairs (Sr$\text{NO}_3^+$, Ca$\text{NO}_3^+$, Mg$\text{NO}_3^+$) occurs, accounting for the relatively stronger sorption of divalent cations at high Na concentration
- Inclusion of carbonate dissolution and precipitation
- Reconcile (if possible/necessary) the batch and column experiments
Two-Site Model for Exchange in Batch Experiments

- Sr:Ca (0.1 N)
- Sr:Ca (0.01 N)
- Sr:Ca (0.001 N)
- Sr:Na (0.1 N)
- Sr:Na (0.01 N)
- Sr:Na (0.001 N)
Exchange Involving Monovalent Ion Pairs?

Can the apparent ionic strength dependence of selectivity coefficients be explained with monovalent ion pairs?

Fit of batch data using exchange of monovalent ion pairs predicts substantial retardation of nitrate breakthrough
Exchange Involving Monovalent Ion Pairs?

No retardation of nitrate observed

\[ XNa^+ + SrNO_3^- \rightleftharpoons XSrNO_3^- + Na^+ \]
## Selectivity Coefficients from Batch Experiments

<table>
<thead>
<tr>
<th></th>
<th>Log Na/Sr</th>
<th>Log Na/Ca</th>
<th>Log Na/Mg</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Site 1 (58%)</strong></td>
<td>0.00</td>
<td>-0.01</td>
<td>-0.11</td>
</tr>
<tr>
<td><strong>Site 2 (42%)</strong></td>
<td>-1.37</td>
<td>-1.19</td>
<td>-0.64</td>
</tr>
</tbody>
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![Graphs showing concentration vs. pore volumes for different conditions.](image-url)
Sr Transport at Hanford 100N

- Strontium contamination near the Columbia River involves seasonal oscillation in groundwater wells
  - Increase in river stage causes intrusion of dilute Columbia River water
  - Lowering of river stage results in return of more concentrated (Ca, Na, Mg) groundwater (although still dilute compared to the Hanford tanks)
Elution of 10 µM Sr in “groundwater” matrix

Switch to Columbia River water with no Sr
Summary and Conclusions

- Scaling effects for reaction rates at the pore scale are minor (at best) due to
  1. Efficiency of diffusive mixing
  2. Slowness of many rates in the subsurface
- Upscaling of rates necessary at larger scales where mixing via diffusion is incomplete (pore network and >)
- Capillary tube experiments ideal for combining simultaneous measurement of solid-solid transformation(s) and effluent chemistry
- X-ray microtomography offers promise for improving estimates of diffusivity and reactive surface area
- The $K_d$ for Sr is quite sensitive to relatively small changes in groundwater chemistry (especially Ca)--Role of ion pairs (SrNO$_3^+$) is minor