Homework No. 4 Solutions

Both problems depend on choices of values & treatment of equations. Some homeworks assessed the magnitude of these issues, which is wise.

(1.) Check stress state in base metal & weld:

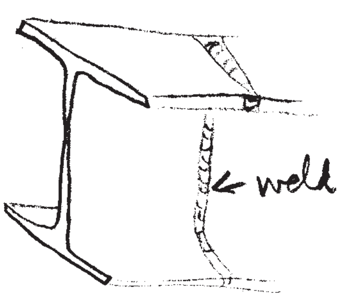
$$2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2 = 1.11 \text{ in. in base metal}$$

$$K_{Ic} \text{ of weld} = \sqrt{\frac{J_{Ic} E}{(1-\nu^2)}} = 47.4 \text{ ksi}\sqrt{\text{in}}$$ $= 49.7$

$$2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2 = 0.69 \text{ in. in weld}$$ (0.76) (assume σ_y in weld $\approx$ 90 ksi)

Thus expect plane strain in each <u>at fracture</u>.

a) Crack was either 0.10 in or 0.30 in. long at failure. Compute a_c for both cases in base metal, and short case in weld:



WELD: $a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_d}\right)^2 = 0.25$ in. σ_d = design σ

(use $K_I = \sigma\sqrt{\pi a}$ for this case)

Thus weld crack could not have been critical size at σ_d.

BASE METAL:

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_d}\right)^2 = 0.39 \text{ in.}$$

Thus neither crack length can
have been critical at failure.

BUT: failure occurred. Check what
$\sigma_a > \sigma_d$ would be needed:

for $a_c = 0.10''$, $\sigma = 107$ ksi

$a_c = 0.30''$, $\sigma = 61.8$ ksi

Both exceed σ_d; for 0.10", exceeds σ_y.

Conclusion: crack did not go unstable in
weld, since a_c too big (even at $\sigma_a = 62$ ksi, $a_c = 0.19''$). Base metal crack could
only be unstable for $\sigma_a > \sigma_d$.

b) If weld was faulty, i.e. caused failure,
crack would go critical there. It looks
instead like it went critical well outside the
weld, so that it is hard to blame the weld
directly. Overweight trucks could have
exceeded σ_d, so it's very relevant here: it
explains how a crack could go critical below
a_c for the σ_d. The question of winter temperature
was not raised in this court case, but could
be relevant if properties are strong functions of
temp. (possible, but not necessarily so) and
if ones quoted are for "70°F" – they could
be minima for worst conditions, i.e. temp.

c) – Check J_{Ic} and K_{Ic} for actual weld, also make sure about temp. question raised in (b).

– Do best poss. job on fracture surface: look for fracture mode, esp. evidence of fatigue (even estimate da/dN), clean as best as poss. for salt corrosion. May see "stretch zone" at onset of instability. check for plasticity (gross) in structure, if $\sigma = 107$ ksi!

d) Clearly, if σ_d is at fault, I-beams should have been heavier or steel stronger (and/or tougher). For present design, enforce load limits or even <u>reduce</u> permissible vehicle loads on bridge. Also could inspect bridge (unlikely and thus undesirable option).

Problem 2

a) $K_I = \sigma\sqrt{\pi a}$

at design stress, $a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{0.6\sigma_y}\right)^2 = 0.0982$ in.

at proof stress, $a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{0.9\sigma_y}\right)^2 = 0.0437$ in.

Permissible crack length = 75% of a_c (design)

$= 0.75\,(0.0982) = 0.0737$ in

$\therefore$ δa for fatigue $= 0.0737 - 0.0437 =$ <u>0.0300 in</u>

Fatigue frequency <u>ν = 4.58</u> cycles / hr (on 24-hr basis)

$$\delta t = \frac{1}{\nu}\left(\frac{\delta a}{da/dN}\right) = \frac{1}{\nu}(\delta N)$$

Vital point: load range is 0.3–0.5 σ_y, not 0–0.5.

$$\frac{da}{dN} = C(\Delta K)^n = C\left(\Delta\sigma\sqrt{\pi a}\right)^n$$

$C = 7\times10^{-12}$ in/cycle $n = 4.2$ $\Delta\sigma = 0.2\,\sigma_y = 36$ ksi

$$\frac{da}{dN} = C(\Delta\sigma\sqrt{\pi})^n(\sqrt{a})^n$$

$$\int_{a_0}^{a_f}\frac{da}{(\sqrt{a})^n} = C(\Delta\sigma\sqrt{\pi})^n\int_0^N dN = C(\Delta\sigma\sqrt{\pi})^n N$$
$$= 2.664\times10^{-4}\,N$$

Integrating,

$$\int_{a_0}^{a_f} a^{-2.1}\,da = -\frac{1}{1.1\,a^{1.1}}\Bigg|_{0.0437}^{0.0737} = -16.01 + 28.45 = 12.44$$

thus $\frac{12.44}{2.664\times10^{-4}} = \delta N = 46{,}695$ cycles

$\frac{\delta N}{\nu} = 1.16$ years $= 424.8$ days $= \delta t$

if ΔK chosen as $0.5\sigma_y$, $\delta \cong 574$ cycles
~ 5 days

Like to know: frequency of loading as accurately as possible; verification of Paris law for this R ratio (R=.6)

b) $0.0737 - 0.050 = 0.0237 \text{ in} = \delta a$

$$\delta N = \left(\frac{-\frac{1}{1.1 a^{1.1}} \Big|_{0.050}^{0.0737}}{C(\Delta\sigma\sqrt{\pi})^{n}} \right) = \frac{-16.01 + 24.53}{2.664 \times 10^{-4}} = 31{,}991 \sim$$

$\delta t = 0.80$ yrs. $= 291$ days

For $\Delta K = 0.5\,\sigma_y$, 450 cycles, ~4 days

Biggest uncertainty: reliability of NDE (cracks might be closed when inspected).

c) Yes $0.3\sigma_y = 54$ ksi > threshold

Tests:

1. determine Cl^- concentration in hydraulic fluid; compare SCC rate with this Cl^- content.
2. Superimposed fatigue could give synergistic effect: should simulate service exactly.

d) δa from (a) is 0.0300 in. = 0.0762 cm.

$$\delta t = \frac{\delta a}{da/dt} = \frac{0.0762}{1 \times 10^{-8}} = \underline{\underline{88 \text{ days}}}$$

Seawater data OK unless concentration effect (see (c)) changes rate

Environment is Cl^- in hydraulic fluid, not water; could be different.

e) $\delta = 0.0762$ cm

$$\delta t = \frac{0.0762}{9.5\times10^{-10}} = \underline{2.54 \text{ yrs}} \overset{926 \text{ days}}{=} \underline{2 \text{ yrs } 6 \text{ mos } 16 \text{ days}}$$

Moderate confidence:

1. can't be Cl^- free!
2. other things in fluid could cause SCC
3. fatigue may radically alter rates

Obviously, should do tests in "Cl^--free" fluid!

But the "Cl^--free" fluid is definitely a good idea (pending SCC or corrosion fatigue data on it)