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Statistical RKR Modeling of Mixed-Mode Fracture in a Brittle Functionally Graded Material

by

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Outline

- LEFM solution and stress intensity factors for FGM's
- Statistical Ritchie-Knott-Rice (RKR) modeling
- Finite element analysis and K-calibration for fracture mechanics sample with modulus gradient
- Calculate effect of gradient slope on
    - predicted fracture toughness, K_{Φ}
    - average kinking direction, α

Singular Crack Tip Fields in an FGM

- The singular stress field retains the strength and form of a homogeneous material [Erdogan,1994]
- As $r \rightarrow 0$ in an FGM with

  $$E(x) = E_o \exp(bx) \qquad n(x) = v_o$$

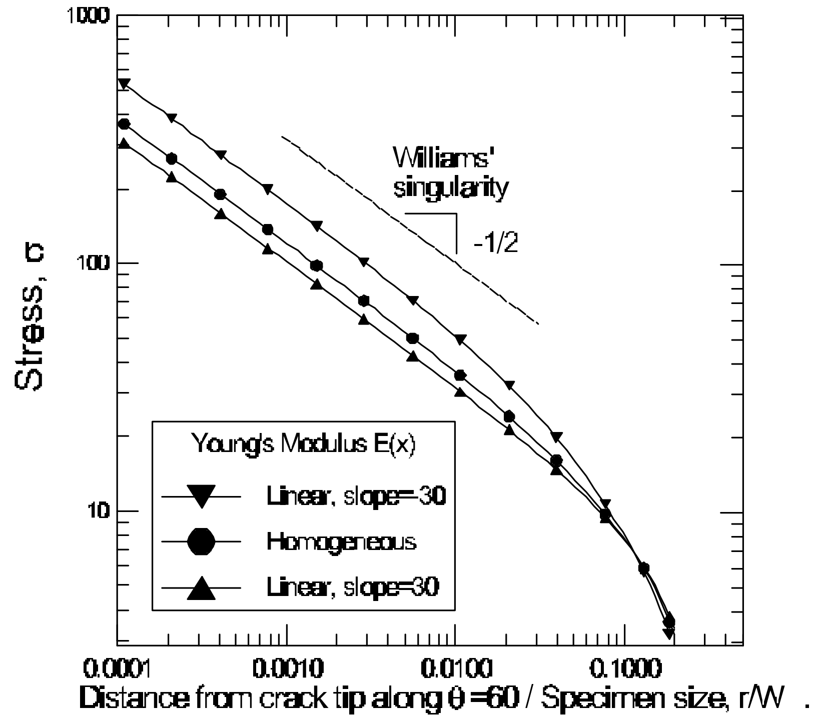
  the stress field varies

$$s_{ij} = \exp(bx)\left[\frac{K_I f_{ij}^{I}(q)}{\sqrt{2p\, r}} + \frac{K_{II} f_{ij}^{II}(q)}{\sqrt{2p\, r}}\right]$$

where K_I = mode-I S.I.F. (tensile mode)

K_{II}= mode-II S.I.F. (shear mode)

- Similar to interface cracks, the K solutions for FGM's depend on the material



Statistical Ritchie-Knott-Rice (RKR) Fracture Model

- The RKR fracture model correlates the onset of fracture with the development of a critical stress at a distance ahead of the crack tip.
- A basis for this behavior is the influence of sampling volume on the measured strength of brittle materials. Using two-parameter Weibull statistics

$$\Phi = 1 - Exp\left[- \int_{vol} \left(\frac{\sigma}{\sigma_o} \right)^m \frac{dV}{V_o} \right]$$

Φ : total failure probability of a part
m : Weibull modulus
σ_o : scaling Weibull stress

- Substituting singular crack tip stress field ($b \leq B$, total sample thickness)

$$\Phi = 1 - \exp\left[-2 \int_0^{\pi} \int_0^{R} \left(\frac{K}{\sigma_o \sqrt{2\pi r}} f_{ij}(\theta) \right)^m \frac{b r dr d\theta}{V_o} \right]$$

- Lin, Evans and Ritchie (1986) use this methodology to describe the fracture behavior of low-toughness steels as a function of temperature

Principal Question

Given:

1) Statistical RKR $\Rightarrow$ Fracture of a brittle material can be calculated as function of stresses *away* from crack tip

2) FGM Crack-tip solution $\Rightarrow$ Stresses are a function of modulus variation

What are the effects of modulus and strength gradients on the toughness and average kink direction of a brittle FGM ?

Crack Tip Modeling

- Φ integral is not defined for m>4 due to the strong singularity in σ^m!
- Lin, et al 1986 integrated the linear elastic stresses outside the plastic zone and nonlinear elastic stresses with a simplified blunting region.
- Here crack is modeled as a slender notch. Integration was performed over the sample, excluding a small near-notch zone with radius $\rho \sim a/10^5$
    - Results were weakly sensitive to the size of this zone
- For a given failure probability, toughness K_Φ can be calculated

$$K_\Phi = \left[\frac{-\ln(1-\Phi)\ V_o}{b\int (\tilde{s}_1 / s_o)^m dA} \right]^{1/m}$$

(with $\tilde{s}_1 = s_1 / K$)

Mean Location and Direction of Fracture

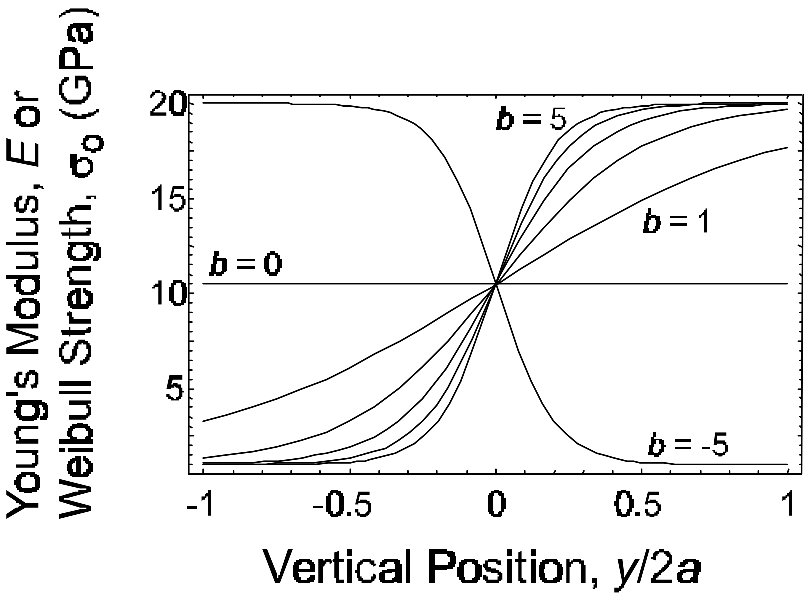
•For the Williams' and HRR crack-tip fields, the most probably distance of fracture initiated, *r** has been calculated for the mode-I case

•Similarly, for mixed-mode loading, determine the average location {*x*,*y*}, {*r*,α} via a weighted Weibull integral

$$\begin{Bmatrix} \bar{x} \\ \bar{y} \end{Bmatrix} = \frac{\int \begin{Bmatrix} x \\ y \end{Bmatrix} \left(\frac{s}{s_o} \right)^m dA}{\int \left(\frac{s}{s_o} \right)^m dA} \qquad \bar{r} = \sqrt{\bar{x}^2 + \bar{y}^2} \qquad \bar{a} = ArcTan(\bar{x}, \bar{y})$$

Procedures

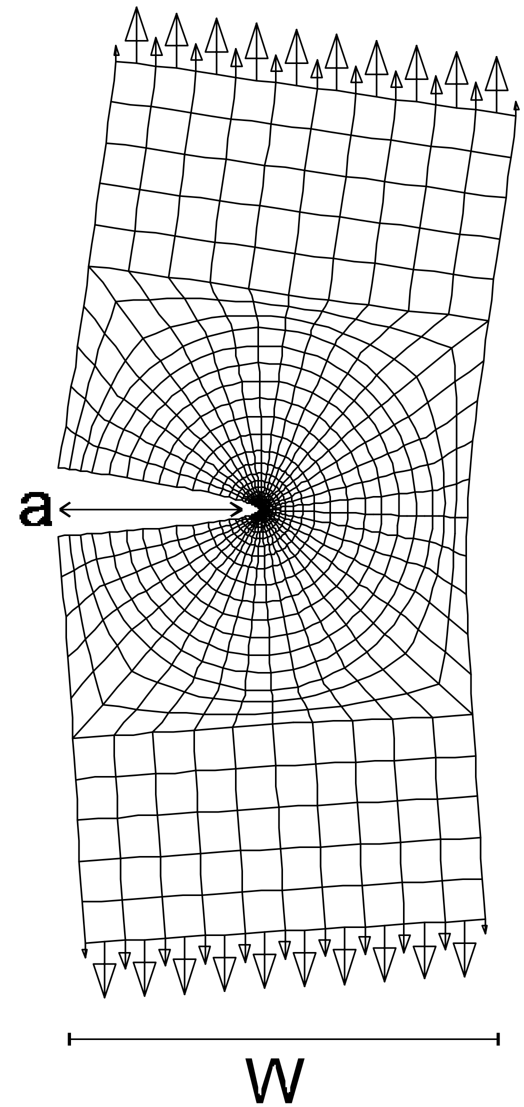
- Two gradient shapes were studied, allowing for a twenty-fold change in properties:
  - $E(x), s_o(x) = b\,x + a$
    - b = [-18, 18] ; a=10.5
  - $E(y), s_o(y) = (a-1)\tanh(b\,y)+a$
    - b = [0, 5] ; a=10.5
- Plane strain; Poisson's ratio, ν=0.3 .
- Calculations were performed for a SEC(T) sample with a single crack length, a/W = 0.5, W = 1
- *K*-calibrations were performed for each gradient considered



Numerical Procedures

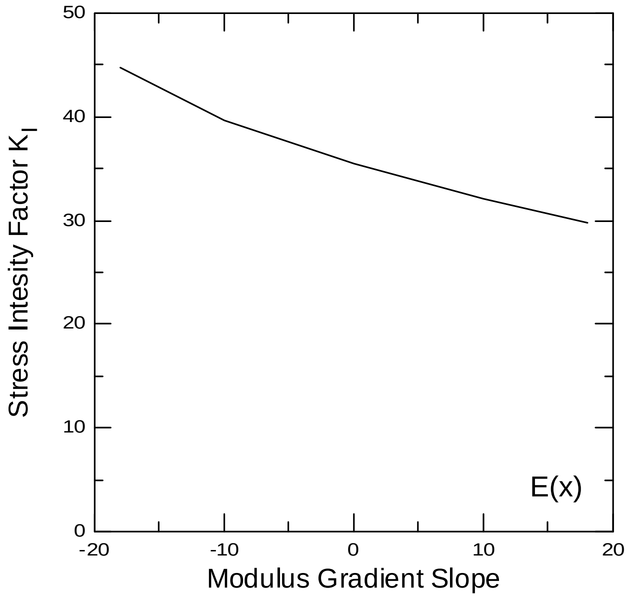
- Finite element code FEAP 4.2 (Zienkiewicz & Taylor, 1987) used with plane-strain linear elastic finite element such that elastic constants were varied quadradically within a single element.
    - element formulation checked against solution of rigid indentation of FGM (Kassir, 1974).
    - crack tip modeled with 40 singular Stern & Becker triangular elements in fan array.
    - 2300 total elements, 9333 nodes.
- Weibull integral was calculated from FEA viz.

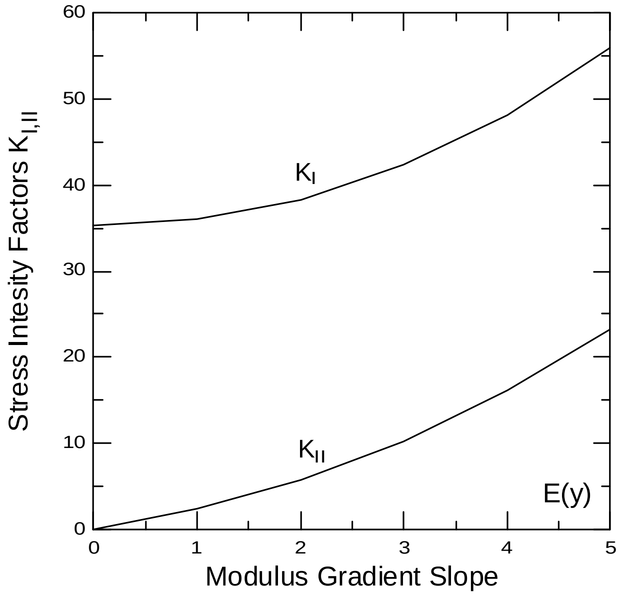
$$\int_{vol} \left(\frac{\sigma}{\sigma_o}\right)^m \frac{dV}{V_o} \approx \frac{b}{\sigma_o^{\,m} V_o} \sum_{j}^{elems} \left[\sum_{i}^{Gauss\ pts} \left(\tilde{\sigma}_1^{\,m}\right)_i J w_i \right]_j$$



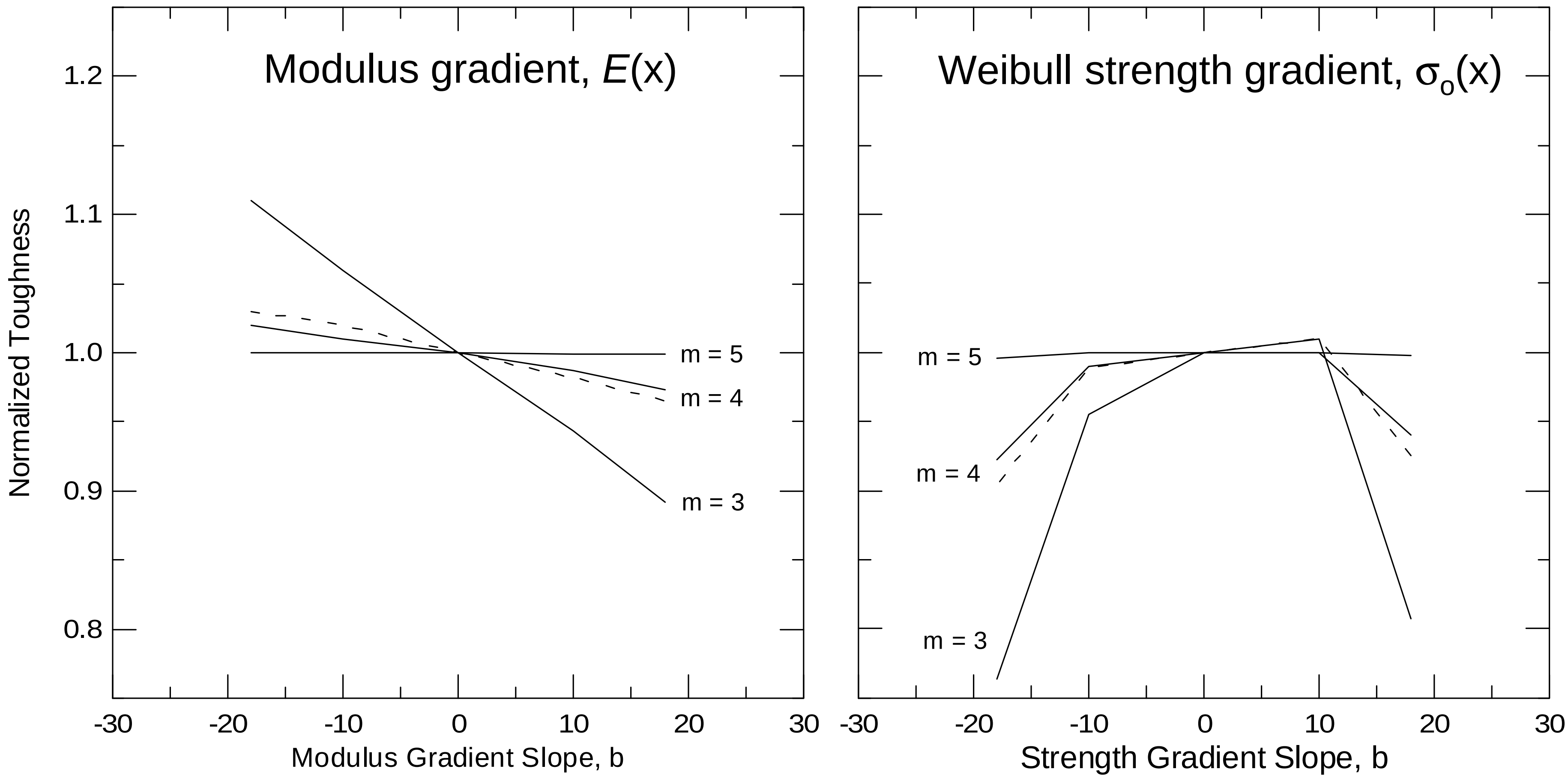
Stress Intensity Factors for $E(x)$ and $E(y)$

- For a homogeneous SEC(T), $K=f(P,a/W)$, independent of modulus. For an FGM $K=f(P,a/W, bW)$ needs to be determined.
- K_I, K_{II} for each gradient obtained by fitting the stresses ahead of the crack.
- The *E(x)* results indicate that the crack tip is shielded when entering stiffer material.
- For comparing the different gradients, failure probabilities can be calculated for the same applied load *P*, or for the same applied *K*.
- *Changing the basis for comparison will reverse the trends observed.*
- For *E(y)* increase in gradient slope increase phase angle





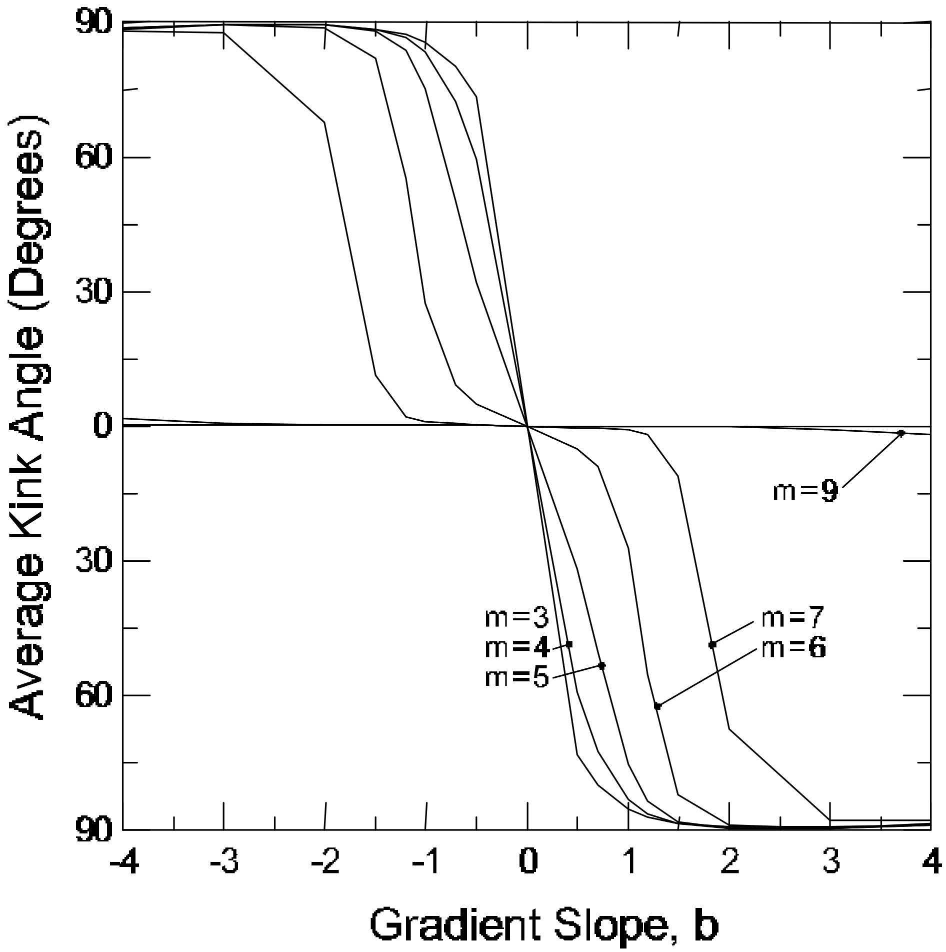
Predicted Fracture Toughness



- Linear gradients in x
- Mode-I loading in all cases

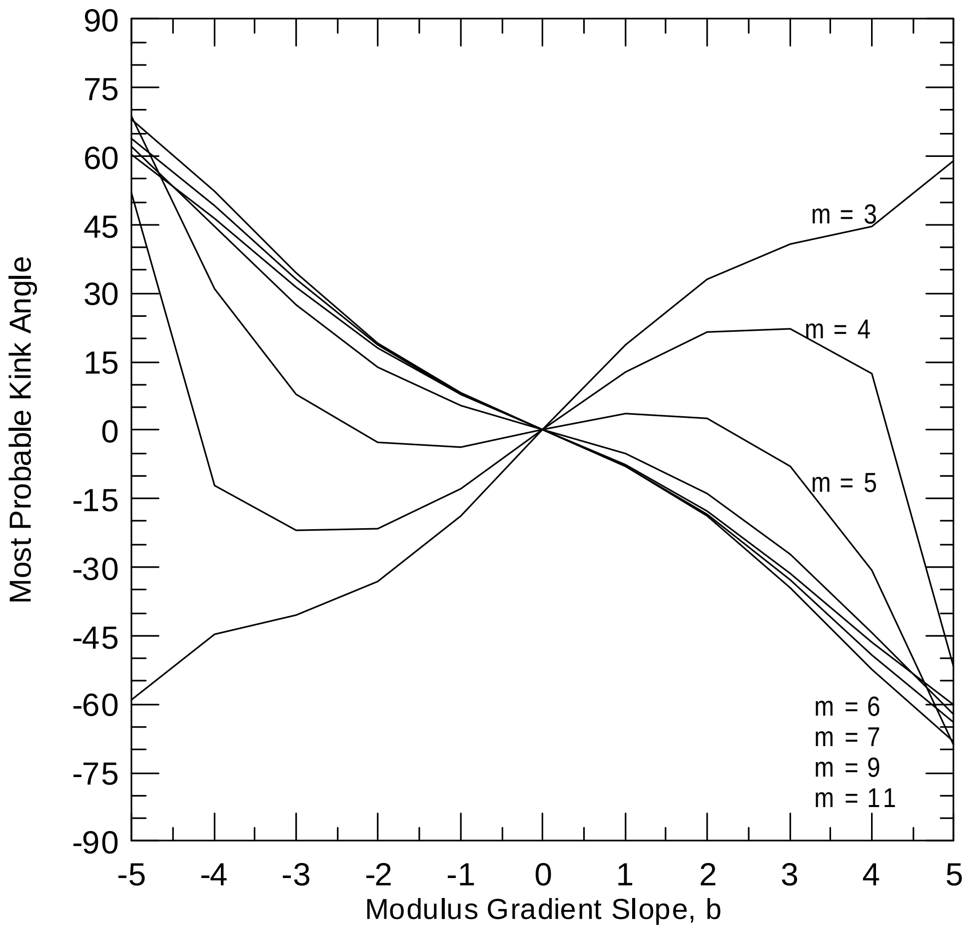
Predicted Kink Angle

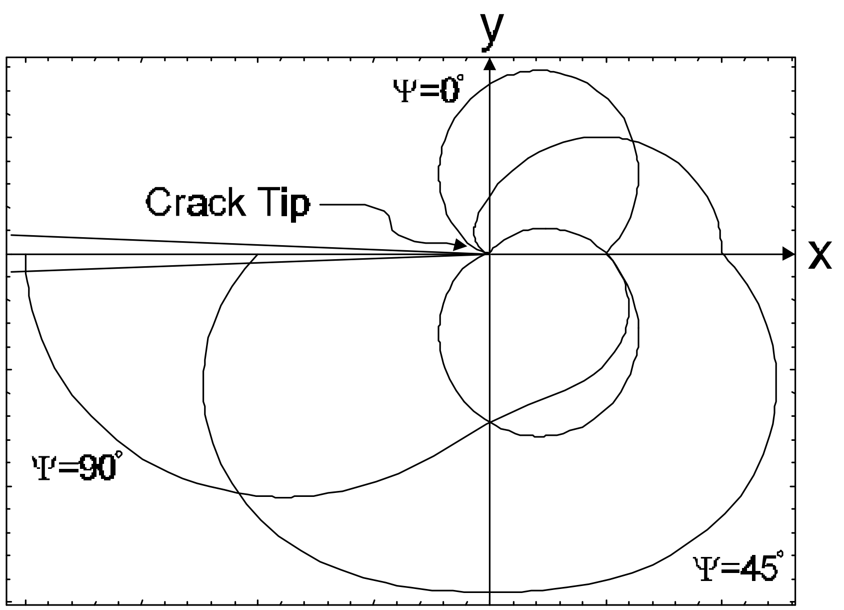
- Gradient in Weibull Strength, $\sigma_o(y)$
- Far-field & near-tip mode-I loading only, K_{II}=0



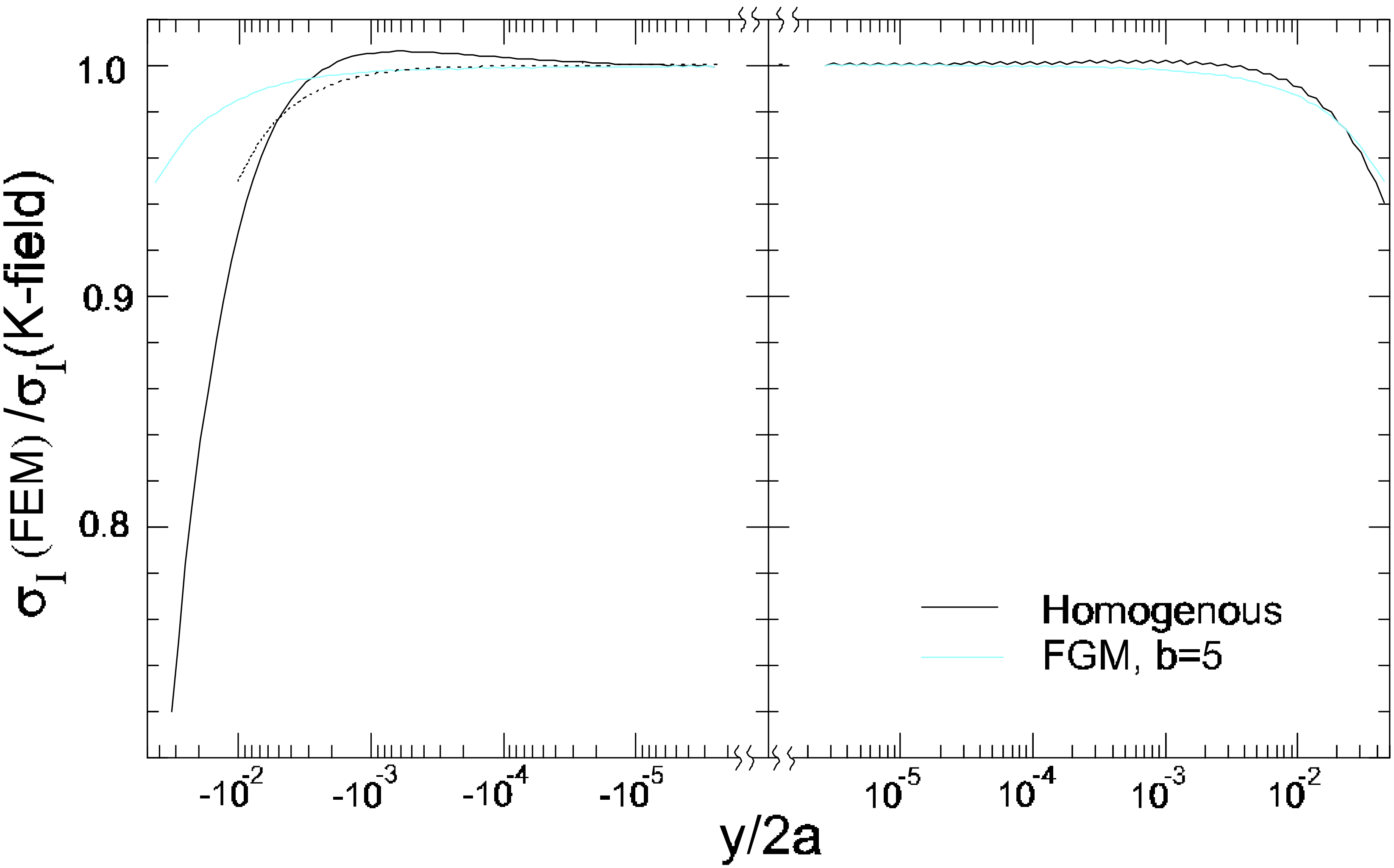
Predicted Kink Angle

- Gradient in modulus, $E(y)$
- Far-field mode-I loading; near-tip mixed mode, K_{I} & K_{II}





Stress Field with Modulus Gradient



Summary

- Finite element calculations indicate stress intensity shielding for cracks in an FMG with a positive modulus slope
- Current model predicts expected fracture toughness will increase for cracks growing into a more compliant material.
- Kinking analysis predicts sharp kinks in FGMs with strength gradients and Weibull moduli, $m<7$
- For FGMs with $E(y)$, nominal mode-I loading results in mixed-mode loading at the crack tip
- For very low Weibull modulus materials, kinking analysis predicts trends opposite to that dictated by near-tip considerations