The Hagedorn Thermostat: stability against multifragmentation

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A system $\mathcal{H}$ with a Hagedorn-like mass spectrum imposes the same temperature to all emitted particles which are then in physical and chemical equilibrium with $\mathcal{H}$ and with each other. The near indifference of $\mathcal{H}$ to fragmentation or coalescence makes this approach relevant to heavy ion and elementary particle collisions alike.

Hagedorn noted that the hadronic mass spectrum (level density) has the asymptotic ($m \to \infty$) form

$$\rho_{\mathcal{H}}(m) \approx \exp\left(m/T_{\mathcal{H}}\right), \tag{1}$$

m is the mass of the hadron and $T_{\mathcal{H}}$ is the parameter (temperature) controlling the the mass spectrum [1, 2].

The MIT bag model [3] produces the same behavior via a constant pressure B of the containing bag [4, 5]. The bag pressure B forces a constant temperature T_B and enthalpy density ϵ, thus the entropy is

$$S = \epsilon V/T_B = m/T_B, \tag{2}$$

V and m are the volume and mass of the bag respectively. This leads to a bag mass spectrum identical to Eq. (1) [4, 5]. This implies the lack of any bag surface energy.

A system $\mathcal{H}$ with a Hagedorn-like spectrum is a perfect thermostat at constant temperature $T_{\mathcal{H}}$ [6]. A perfect thermostat is indifferent to the transfer of any portion of its energy to any parcel within itself. It is at the limit of phase stability and the internal energy density fluctuations are maximal. It does not matter whether the thermostat is one large bag or fragmented in an arbitrary number of smaller bags or, equivalently, it is a system of hadrons with a spectrum given by Eq. (1).

To see this consider $\mathcal{H}$ coupled to an ideal vapor [6] with a free vapor particle mass m. The microcanonical level density of the vapor with kinetic energy ε is

$$\rho_{\rm vapor}(\varepsilon) = \frac{V^N}{N!\left(\frac{3}{2}N\right)!}\left(\frac{m\varepsilon}{2\pi}\right)^{\frac{3}{2}N}, \tag{3}$$

V is is the volume. The full microcanonical partition is

$$\begin{aligned} \rho_{\rm tot}(E,\varepsilon) &= \rho_{\mathcal{H}}(E-\varepsilon)\rho_{\rm vapor}(\varepsilon) \\ &= \frac{V^N}{N!\left(\frac{3}{2}N\right)!}\left(\frac{m\varepsilon}{2\pi}\right)^{\frac{3}{2}N} e^{\frac{E-mN-\varepsilon}{T_{\mathcal{H}}}}. \end{aligned} \tag{4}$$

This gives the most probable kinetic energy per particle as $\varepsilon/N = 3T_{\mathcal{H}}/2 \equiv \varepsilon_{\mathcal{H}}$ and the most probable particle density of the vapor *independent of* V as $N/V = (mT_{\mathcal{H}}/2\pi)^{3/2}\exp\left(-m/T_{\mathcal{H}}\right) \equiv n_{\mathcal{H}}$. Using $\varepsilon_{\mathcal{H}}$ and $n_{\mathcal{H}}$ gives the most probable value of the system's level density $\rho^*_{\rm tot}(E,\varepsilon) \approx \exp\left[S^*\right]$, where the entropy is $S^* = E/T_{\mathcal{H}} + N$. Differentiating $\rho^*_{\rm tot}(E,\varepsilon)$ with respect to m and using $n_{\mathcal{H}}$ gives

$$\partial \ln \rho^*_{\rm tot}(E,\varepsilon)/\partial m = N\left[3/2m - 1/T_{\mathcal{H}}\right]. \tag{5}$$

For $N \neq 0$ the most probable value of $\rho^*_{\rm tot}(E,\varepsilon)$ is for $m = \frac{3}{2}T_{\mathcal{H}} \equiv m_{\mathcal{H}}$. Since all intrinsic statistical weights in $\rho^*_{\rm tot}(E,\varepsilon)$ are factored into a single $\mathcal{H}$, the system breaks into fragments with $m_{\mathcal{H}}$ except for one whose mass is set by mass/energy conservation. Substituting $\varepsilon_{\mathcal{H}}$ and $m_{\mathcal{H}}$ into $n_{\mathcal{H}}$ gives the vapor concentration as

$$N/V = (3/4\pi e)^{3/2}\, T^3_{\mathcal{H}}. \tag{6}$$

The density of the vapor of nonrelativistic particles acquires the form typical of the ultrarelativistic limit.

If the value of $m_{\mathcal{H}}$ does not exist, then the most probable value of $\rho^*_{\rm tot}(E,\varepsilon)$ corresponds to the mass m^* nearest to $m_{\mathcal{H}}$ and $n_{\mathcal{H}}(m^*)$. In terms of hadron spectroscopy the pion mass maximizes the level density $\rho^*_{\rm tot}(E,\varepsilon)$.

If we require $\mathcal{H}$ to completely fragment into equal mass fragments all with translational degrees of freedom, then

$$\rho_{\rm tot}(E,\varepsilon) = \frac{V^N}{N!}\left(\frac{mT_{\mathcal{H}}}{2\pi}\right)^{\frac{3}{2}N} e^{\frac{E}{T_{\mathcal{H}}}}, \tag{7}$$

where we used $\varepsilon_{\mathcal{H}}$ and the Stirling formula for $\left(\frac{3}{2}N\right)!$. Equation (7) shows that all the Hagedorn factors collapse into a single one with the m-independent argument E. Maximization of (7) with respect to m leads to

$$\partial \ln \rho_{\rm tot}(E,\varepsilon)/\partial m = 3N/2m = 0\,, \tag{8}$$

which is consistent with $N = 1$ and $m = E$, namely a single Hagedorn particle with all the available mass.

This illustrates the indifference of $\mathcal{H}$ toward fragmentation. Of course $\varepsilon_{\mathcal{H}}$ gives directly the mass distribution of the Hagedorn fragments under the two conditions discussed above. These results justify the assumption of the canonical formulation of the statistical hadronization model that smaller clusters appear from a single large cluster [7].

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