The Hagedorn Thermostat: the unique temperature $T_{\mathcal{H}}$

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A system $\mathcal{H}$ with a Hagedorn-like mass spectrum can sustain only the unique temperature $T_{\mathcal{H}}$ encoded in the spectrum itself. $\mathcal{H}$ imposes the same temperature to all emitted particles. This may explain the recurring temperature observed in several experiments.

Hagedorn noted that the hadronic mass spectrum (level density) has the asymptotic ($m \to \infty$) form

$$\rho_{\mathcal{H}}(m) \approx \exp\left(m/T_{\mathcal{H}}\right), \tag{1}$$

m is the mass of the hadron and $T_{\mathcal{H}}$ is the parameter (temperature) controlling the the mass spectrum [1, 2].

The MIT bag model [3] produces the same behavior via a constant pressure B of the containing bag [4, 5]. In vacuum the bag pressure B forces a constant temperature T_B and enthalpy density ϵ, thus the entropy is

$$S = \epsilon V/T_B = m/T_B, \tag{2}$$

V and m are the volume and mass of the bag respectively. This leads to a bag mass spectrum identical to Eq. (1) [4, 5]. This implies the lack of any bag surface energy.

Many experiments with high energy elementary particle collisions on different systems indicate a constant temperature characterizing both chemical and physical equilibrium [6–8]. We explore the connection of these empirical temperatures with the Hagedorn temperature $T_{\mathcal{H}}$ and the bag temperature T_B.

A system $\mathcal{H}$ possessing a Hagedorn-like spectrum, characterized by an entropy of the form (2), not only has a unique microcanonical temperature $T_{\mathcal{H}}$

$$T_{\mathcal{H}} = \left(dS/dE\right)^{-1} = T_B, \tag{3}$$

but also imparts this same temperature to any other system to which $\mathcal{H}$ is coupled. $\mathcal{H}$ is a perfect thermostat with the constant temperature $T_{\mathcal{H}}$.

To demonstrate this we couple $\mathcal{H}$ to a one dimensional harmonic oscillator and use a microcanonical treatment. The unnormalized probability $P(\varepsilon)$ for finding an excitation energy ε in the harmonic oscillator out of the system's total energy E is

$$\begin{aligned} P(\varepsilon) &\sim \rho_{\mathcal{H}}(E-\varepsilon)\,\rho_{\rm osc}(\varepsilon) \\ &= \exp\left(\frac{E-\varepsilon}{T_{\mathcal{H}}}\right) = \rho_{\mathcal{H}}(E)\exp\left(-\frac{\varepsilon}{T_{\mathcal{H}}}\right). \end{aligned} \tag{4}$$

For a one dimensional harmonic oscillator $\rho_{\rm osc}$ is a constant. The energy spectrum of the oscillator is canonical up to the upper limit $\varepsilon_{max} = E$ with an inverse slope (temperature) of $T_{\mathcal{H}}$ independent of E. The mean value of the energy of the oscillator is:

$$\overline{\varepsilon} = T_{\mathcal{H}}\left[1 - \frac{E/T_{\mathcal{H}}}{\exp\left(E/T_{\mathcal{H}}\right)-1}\right]. \tag{5}$$

Thus in the limit that $E \to \infty$: $\overline{\varepsilon} \to T_{\mathcal{H}}$, i.e. no temperature other that $T_{\mathcal{H}}$ is admitted.

Similarly, consider a vapor of $N \gg 1$ non-interacting particles of mass m coupled to $\mathcal{H}$. The microcanonical level density of the vapor with kinetic energy ε is

$$\rho_{\rm vapor}(\varepsilon) = \frac{V^N}{N!\left(\frac{3}{2}N\right)!}\left(\frac{m\varepsilon}{2\pi}\right)^{\frac{3}{2}N}, \tag{6}$$

where V is is the volume. The microcanonical partition of the total system is

$$\begin{aligned} \rho_{\rm total}(E,\varepsilon) &= \rho_{\mathcal{H}}(E-\varepsilon)\rho_{\rm vapor}(\varepsilon) \\ &= \frac{V^N}{N!\left(\frac{3}{2}N\right)!}\left(\frac{m\varepsilon}{2\pi}\right)^{\frac{3}{2}N} e^{\frac{E-mN-\varepsilon}{T_{\mathcal{H}}}}. \end{aligned} \tag{7}$$

The distribution of the vapor is exactly canonical up to $\varepsilon_{max} = E$, if the particles are independently present, or $\varepsilon_{max} = E - mN$, if the particles are generated by $\mathcal{H}$. In either case, the temperature of the vapor is always $T_{\mathcal{H}}$.

At fixed N the maximum of $\rho_{\rm total}(E,\varepsilon)$ with respect to ε gives the most probable kinetic energy per particle:

$$\frac{\partial\rho_{\rm total}(E,\varepsilon)}{\partial\varepsilon} = \frac{3N}{2\varepsilon} - \frac{1}{T_{\mathcal{H}}} = 0 \quad\Rightarrow\quad \frac{\varepsilon}{N} = \frac{3}{2}T_{\mathcal{H}}, \tag{8}$$

provided that $E \geq mN + \frac{3}{2}NT_{\mathcal{H}}$. For $mN < E < mN + \frac{3}{2}NT_{\mathcal{H}}$, the most probable value of the kinetic energy per particle is $\frac{\varepsilon}{N} = \frac{E}{N} - m < \frac{3}{2}T_{\mathcal{H}}$; for $E \leq mN$, $\frac{\varepsilon}{N} = 0$. $T_{\mathcal{H}}$ is the sole temperature characterizing the distribution up to the microcanonical cut-off, which may be above or below the maximum of the distribution, since the form of $\rho_{\rm total}(E,\varepsilon)$ is independent of E.

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