Chapter 6 Nuclear Energy Levels

The nucleus, like the atom, has discrete energy levels whose location and properties are governed by the rules of quantum mechanics. The locations of the excited states differ for each nucleus. The excitation energy, E_x, depends on the internal structure of each nucleus. Each excited state is characterized by quantum numbers that describe its angular momentum, parity, and isospin (see chapter 5). Figure 6-1 shows a few of the excited states of the ^{12}C nucleus.

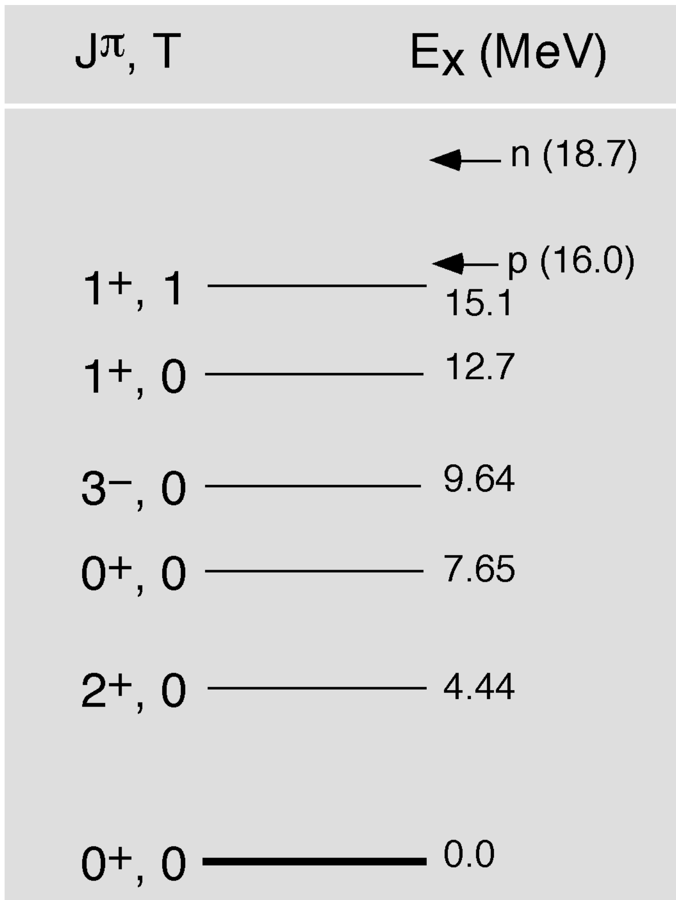

Fig. 6-1. Energy level diagram of some of the excited states of the ^{12}C nucleus. The angular momentum (*J*), parity (*P*), and isospin (*T*) quantum numbers of the states are indicated on the left using the notation J^P. P and n respectively at the top of the diagram indicate the separation energies for a proton and a neutron.

The angular momentum quantum number, *J*, is the integer or half-integer that is the measure of the total angular momentum of the energy state in units of $\hbar$ (Planck's constant h divided 2π)

$$\text{angular momentum} = J\ \hbar .$$

The parity, *P*, of a nuclear energy level is a statement about what the nuclear structure of the state would look like if the spatial coordinates of all the nucleons were reversed. $P = +$ means the reversed state would look the same as the original; $P = -$ means the reversed state differs from the original. The isospin (projection) quantum number, *T*, is an integer or half-integer that measures a property that results if neutron and proton coordinates were interchanged. Figure 6-1 shows these quantum numbers for each excited state in the notation J^P, T. These quantum numbers are results of the basic symmetries of the underlying force law that governs the binding of nucleons in a nucleus. They determine how an excited state will decay into another state in the same nucleus (gamma decay) or into a specific state in a different nucleus (beta or alpha decay).

Analyzing the interactions among many nucleons to calculate the energy levels and their properties is a complicated mathematical task. Instead, nuclear scientists have developed several nuclear models that simplified the description the nucleus and the mathematical calculations. These simpler models still preserve the main features of nuclear structure.

The Shell Model

One such model is the Shell Model, which accounts for many features of the nuclear energy levels. According to this model, the motion of each nucleon is governed by the average attractive force of all the other nucleons. The resulting orbits form "shells," just as the orbits of electrons in atoms do. As nucleons are added to the nucleus, they drop into the lowest-energy shells permitted by the Pauli Principle, which requires that each nucleon have a unique set of quantum numbers to describe its motion.

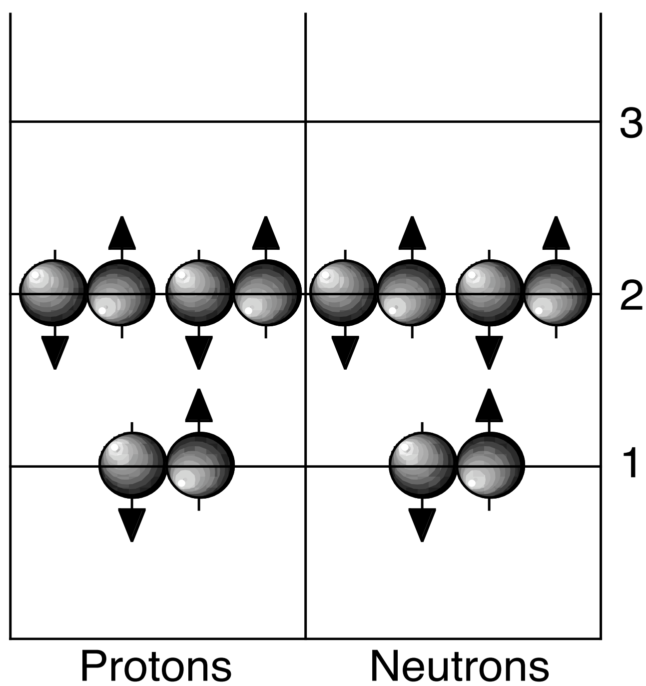

Fig. 6-2. Shell model energy diagram of the orbitals of protons and neutrons in ^{12}C ground state. The arrows on individual nucleons denote its spin state.

When a shell is full (that is, when the nucleons have used up all of the possible sets of quantum number assignments), a nucleus of unusual stability forms. This concept is similar to that found in an atom where a filled set of electron quantum numbers results in an atom with unusual stability—an inert gas. When all the protons or neutrons in a nucleus are in filled shells, the number of protons or neutrons is called a "magic number." Some of the magic numbers are 2, 8, 20, 28, 50, 82, and 126. For example, ^{116}Sn has a magic number of protons (50) and ^{54}Fe has a magic number of neutrons (28). Some nuclei, for example ^{40}Ca and ^{208}Pb, have magic numbers of both protons and neutrons; these nuclei have exceptional stability and are called "doubly magic." Magic numbers are indicated on the chart of the nuclides.

Filled shells have a total angular momentum, *J*, equal to zero. The next added nucleon (a valence nucleon) determines the *J* of the new ground state. When nucleons (singly or in pairs) are excited out of the ground state they change the angular momentum of the nucleus as well as its parity and isospin projection quantum numbers. The shell model describes how much energy is required to move nucleons from one orbit to another and how the quantum numbers change. Figure 6-2 shows an energy diagram of the two filled shells of the ground state of ^{12}C. Promotion of a nucleon or a pair of nucleons to an unfilled shell puts the nucleus into one of the excited states shown in Fig. 6-1.

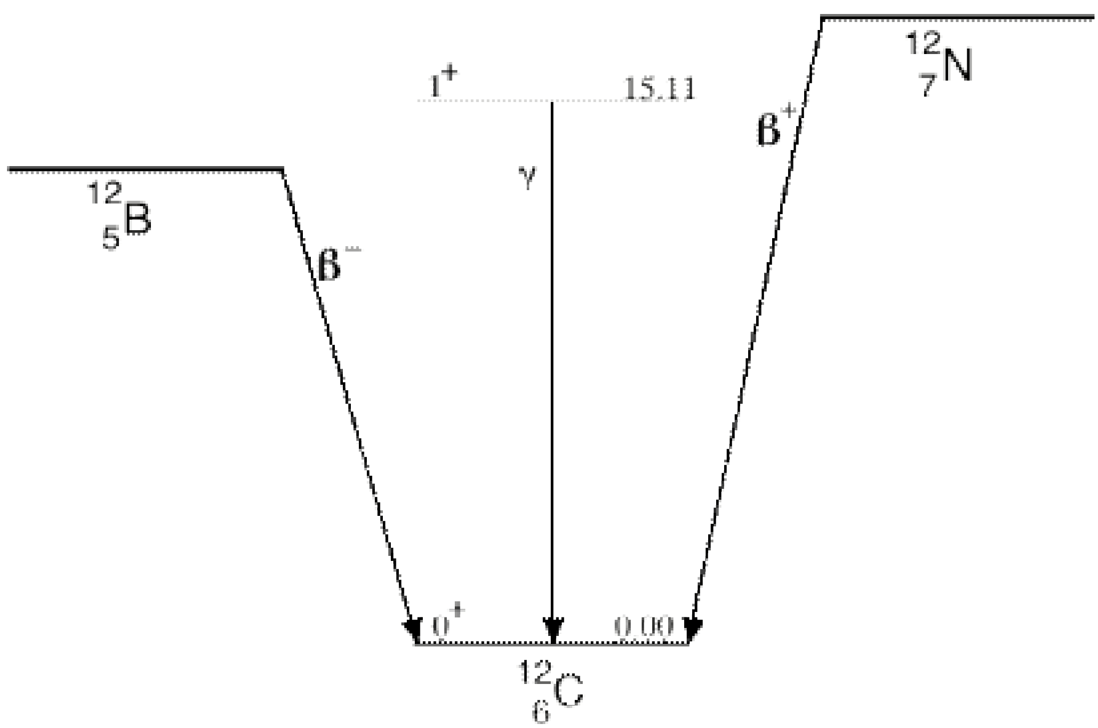

Fig. 6-3. Transitions between discrete energy levels in A = 12 nuclei. The energy is given in MeV.

Excited nuclear states decay to more stable states, *i.e.*, more stable nucleon orbitals. Measuring transition rates between nuclear energy levels requires specialized alpha, beta, and gamma detectors and associated electronic circuitry to precisely determine the energy and half-life of the decay. Quantum mechanics and shell-model theory permit nuclear scientists to compute the transition probability (rate of decay) between nuclear states. For nuclei whose structure can be described by a small number of valence nucleons outside filled shells, the Shell Model calculations agree very well with measured values of spin and parity assignments and transition probabilities.

An example of transitions between a nucleus's energy levels is shown in Fig. 6-3. The ground states of ^{12}B (5 protons, 7 neutrons) and ^{12}N (7 protons, 5 neutrons) are related to each other and to the 15.1 MeV state in ^{12}C. Each has a nucleon in the third energy level shown in Fig. 6-2, each has the quantum number 1^+, 1, and each decays to the ground state of carbon. The boron and nitrogen beta decay emit an e^- and e^+ respectively. The 15.1 MeV state in carbon decays by gamma emission. The shell model calculations of these transition probabilities agree quite well with the measured rates even though different decay mechanisms are at work in each case. Nuclear theorists keep refining the shell model to understand the details of nuclear structure and to make that knowledge available for applications in nuclear technology.

The Collective Model

In addition to individual nucleons changing orbits to create excited states of the nucleus as described by the Shell Model, there are nuclear transitions that involve many (if not all) of the nucleons. Since these nucleons are acting together, their properties are called

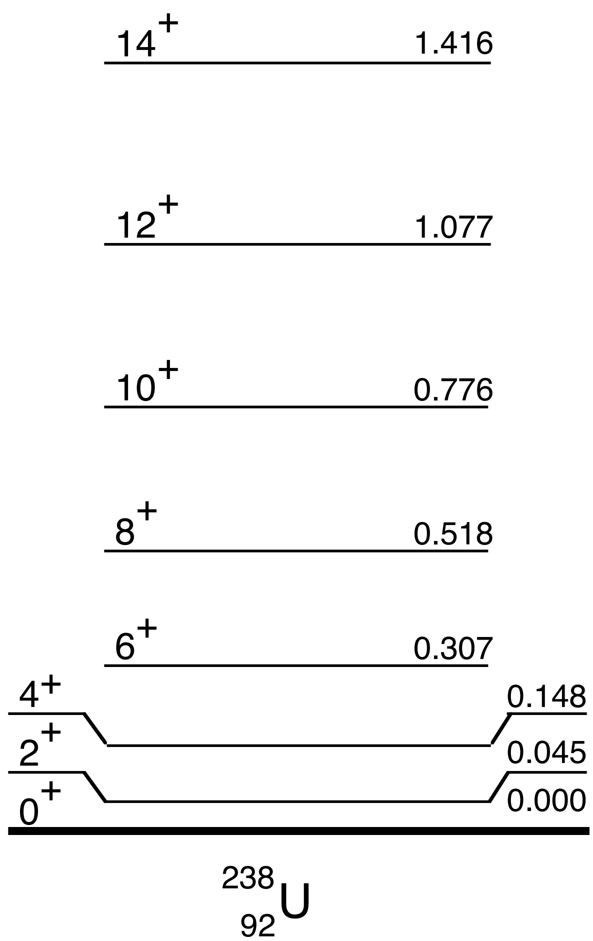

Fig. 6-4. Rotational states of ^{238}U. The energy is given in MeV.

collective, and their transitions are described by a Collective Model of nuclear structure. High-mass nuclei have low-lying excited states that are described as vibrations or rotations of nonspherical nuclei. Many of these collective properties are similar to those of a rotating or vibrating drop of liquid, and in its early development the Collective Model was called the Liquid-Drop Model. The first important application of the Liquid-Drop model was in the analysis of nuclear fission, in which a massive nucleus splits into two lower-mass fragments. The Liquid Drop Model calculates an energy barrier to fission as a sum of the repulsive Coulomb forces between the protons of the nucleus and the attractive surface tension of the skin of the "liquid drop" nucleus. If the barrier is low enough the nucleus might fission spontaneously. For higher barriers, it takes a nuclear reaction to induce fission.

Figure 6-4 shows the energy levels of ^{238}U. The quantum numbers, level spacings, and gamma ray transition probabilities identify these levels as rotational states of a nonspherical nucleus. Nuclei showing collective properties are usually those with many valence nucleons, that is, those with proton or neutron numbers that are far from filled shells. As with the Shell Model, the Collective Model permits the calculation of spin-parity assignments and transition probabilities that are in good agreement with the measured properties of collective nuclei.

Measuring Energy Levels

Using accelerators to make nuclear reactions (see Chapter 7), scientists can create nuclei which have very high angular momentum. Nuclei respond to this rotation, which can be as fast as a hundred billion billion revolutions per second, in a rich and varied way. These nuclei lose some of their excitation energy and almost all of the initial angular momentum by the emission of gamma rays. The gamma ray flash is finished in less than 10^{-9} seconds, during which 30 or more gamma rays can be emitted.

A number of preferred pathways in the de-excitation process occur. They relate to favorable arrangements of protons and neutrons and can often be associated with specific symmetries or nuclear shapes. If a sufficient fraction of the decay flows down a particular quantized pathway or band, then the associated structure becomes observable and can be studied in detail. Scientists have recently built arrays of over 100 gamma ray detectors to study the details of some of these rare pathways. One such array is Gammasphere, shown in Figure 6-5. Gammasphere was built by groups from Lawrence Berkeley National Laboratory, Argonne National Laboratory, and Oak Ridge National Laboratory. It was used at Berkeley Lab's 88-Inch Cyclotron for two years before being moved to Argonne in 1997. During its tenure in Berkeley, many exciting discoveries were made including:

1. <u>*Details of superdeformation in nuclei.*</u> Superdeformation occurs when quantum shell effects help stabilize a football shape (2:1 axis ratio) in certain nuclei. Superdeformed nuclei, prevalent in several regions of the chart of the nuclides, have been found to display some amazing properties.

2. *Identical Bands*. Scientists have discovered that sequences of ten or more identical photons are associated with different bands in neighboring nuclei. This comes as a great surprise; it has long been believed that the gamma-ray emission spectrum for a specific nucleus represents a unique fingerprint. Explaining identical bands is now in the hands of shell model theoreticians.

3. *Magnetic Rotation*. Magnetic rotation occurs in nearly spherical nuclei. It is characterized by sequences of gamma rays reminiscent of collective rotational bands but with a quite different character. Namely, each photon carries off only one (rather than two) units of angular momentum and couples to the magnetic rather than electric properties of the nucleons. This is a new form of quantal rotor that is not fully understood at present.

Figure 6-5. One hemisphere of the Gammasphere detector array.

Gammasphere spent several years at Argonne National Laboratory where it was used for a period of two years to investigate the structure of nuclei far from the line of stability. In late 2000, Gammasphere moved back to Berkeley Lab where it is once again being used for a variety of experiments in high-spin physics.

Web Sites:

Gammasphere
http://www-gam.lbl.gov/ — This web site describes the science behind Gammasphere. You can find many pictures of this apparatus. There is also a link to the research done at Argonne.