The turbulent wall jet: A triple-layered structure and incomplete similarity

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We demonstrate using the high-quality experimental data that turbulent wall jet flows consist of two self-similar layers: a top layer and a wall layer, separated by a mixing layer where the velocity is close to maximum. The top and wall layers are significantly different from each other, and both exhibit incomplete similarity, i.e., a strong influence of the width of the slot that had previously been neglected.

turbulence | turbulent jets | scaling | power law

Turbulent wall jets have many practical uses and have attracted the attention of many experimentalists. There have been many attempts to find scaling laws for these flows, i.e., to find dimensionless coordinates in which the velocity distributions for the various cross-sections collapse to a single curve.

In the present work, we show that earlier attempts to find scaling laws were based on an erroneous understanding of the flow structure near the wall, originating from a lack of resolution near the wall. By using the high-quality experimental data of Karlsson *et al.* (1) we demonstrate that turbulent wall jets possess a more complicated structure than previously thought. We find that wall jet flows consist of two self-similar layers: a top layer and a wall layer, separated by a mixing layer where the velocity is close to the maximum. Most important is that the scaling laws in the top and wall layers are substantially different. Both exhibit incomplete similarity, i.e., a strong influence of the width of the slot that had previously been neglected.

The general shape of an apparatus that produces wall jet flow is presented in Fig. 1*A*; in Fig. 1*B* we show schematically the distribution of mean longitudinal velocity. The flow is as follows. A turbulent jet comes out of a slot. The width of the slot is d, and the momentum flux per unit thickness of the slot is J. At large distances from the slot, the fluid is at rest. Unlike the mixing in a free jet, the mixing here is substantially influenced by the wall as well as non-symmetric.

At a distance from the slot, large in comparison with the slot width d but small in comparison with the overall size of the set-up H, an intermediate-asymptotic flow structure is formed, which has been the main object of interest for experimentalists; indeed, many researchers, starting with Prandtl (2) and Tollmien (3), contributed to its investigation (1, 4–12).

Scaling laws of the form

$$\frac{u}{u_{\max}} = f\left(\frac{y}{y_{1/2}}\right) \quad [1]$$

have been proposed for describing this structure, where $u_{\max}$ is the maximum velocity at a given section x, and $y_{1/2}$ is the coordinate, which also depends on x, where the mean velocity is equal to one half of the maximum velocity; this coordinate has always been taken to be above the point where the maximum velocity is reached, and indeed in the majority of experiments the resolution close to the wall has been insufficient to determine the coordinate under the maximum where one half of the maximum mean velocity is also achieved, though this coordinate obviously exists.

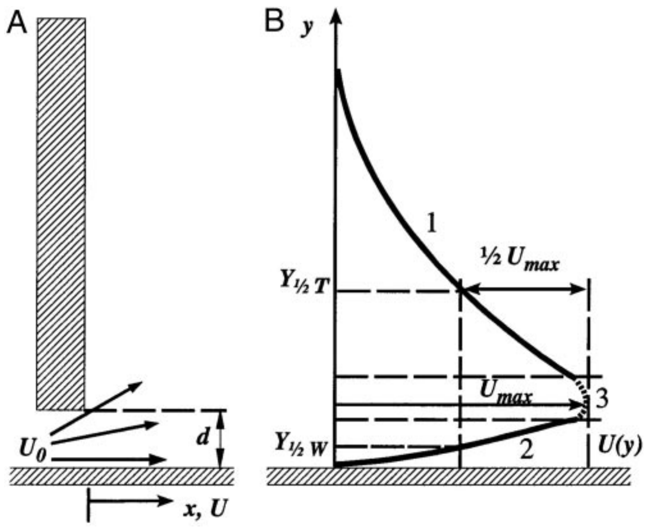

Fig. 1. The schematic structure of the wall jet flow. (*A*) The apparatus that produces a wall jet. (*B*) The structure of wall jet flow. 1, top self-similar layer; 2, wall self-similar layer; 3, mixing layer where the velocity is close to maximum.

To understand the flow, we performed advanced similarity analysis, using the digitized data of high-quality experiments by Karlsson and coworkers (1, 11, 12) included in the ERCOFTAC Classic Database. We found that the situation is more complicated than previously assumed.

In particular, we came to the conclusion that a single self-similar structure in the wall jet, to which one can apply the scaling law, does not exist. Instead, we found that wall jet flow consists of two self-similar flow layers described by significantly different scaling laws and separated by a mixing layer where the velocity is close to the maximum. The scaling laws in both self-similar layers reveal an incomplete similarity, so that the influence of the slot width remains.

Similarity Analysis of Wall Jet Flow

The phenomenon under consideration has the following governing parameters: d, width of the slot; $[d] = L$ (the square brackets denote the dimension of the object in the brackets; L is a dimension of length); H, a characteristic length size of the set-up, $[H] = L$; y, the distance of the observation point from the wall; x, the longitudinal coordinate of the observation point, reckoned from a given origin; $[x] = [y] = L$; J, momentum flux through unit thickness of the slot; $[J] = M/T^2$, ρ = fluid density; $[\rho] = ML^{-3}$; ν = fluid kinematic viscosity, $[\nu] = L^2T^{-1}$; M and T are the dimensions of mass and time, respectively.

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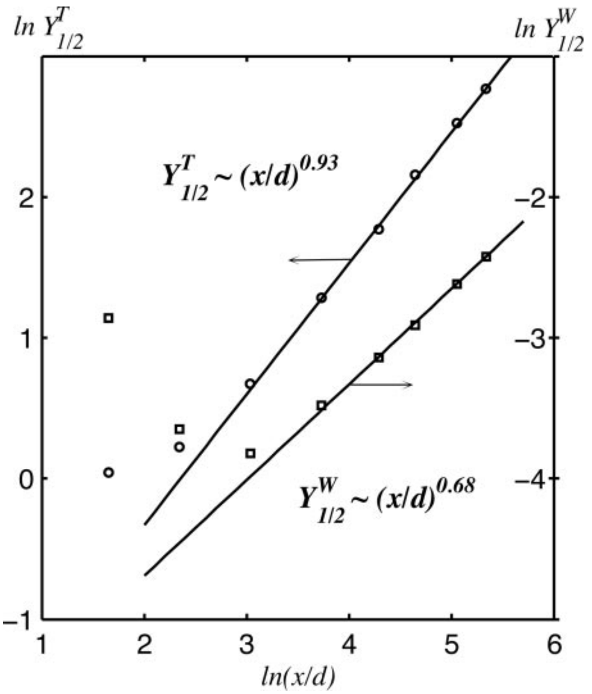

Fig. 2. The lines $y^T_{1/2}(x)$ and $y^W_{1/2}(x)$ where the velocity is equal to one half of the maximum asymptotically approach the two different scaling laws.

Four dimensionless parameters can be formed from the governing parameters

$$\Pi_1 = \frac{y}{d}, \quad \Pi_2 = \frac{x}{d}, \quad \Pi_3 = \sqrt{\frac{Jd}{\rho}} \cdot \frac{1}{\nu}, \quad \Pi_4 = \frac{H}{d}. \qquad [2]$$

Every dimensionless property of wall jet flow can be represented as a function of these parameters. With the large Π_4 characteristic of existing high-quality set-ups, it is natural to assume complete similarity in this parameter so that the value of H is immaterial. The parameter Π_3 is an analog of the Reynolds number; we will denote it by *Re*.

At large distances from the nozzle, the parameter $\Pi_2 = x/d$ is large. However, a simple assumption of complete similarity in this parameter at $\Pi_2 \gg 1$ does not work, in contrast to the situation with flow in pipes and boundary layers. The experiments show (see below) that at large *Re* there exists an intermediate region of distances from the nozzle

$$d \ll x \ll H,$$

where there exists a self-similar structure, but this similarity is incomplete.

Analysis of Experimental Data and Basic Hypotheses

We used the experimental data obtained by Karlsson and coworkers (1), available in digital form in the ERCOFTAC Classic Database. First, we noted the obvious (see Fig. 1*B*): the longitudinal velocity distribution of the wall jet flow has not one but two ordinates where the mean velocity has the value $\frac{1}{2}u_{max}$; we denote them by $y^T_{1/2}$ (the one above the maximum) and $y^W_{1/2}$ (the one nearer the wall). Processing the data (12) in bilogarithmic coordinates, we found that for both of them the scaling laws

$$y^i_{1/2} = A_i d^{1-\beta_i} x^{\beta_i}, \qquad [3]$$

where $i = T$ corresponds to the top layer and $i = W$ to the bottom one, are established after a rather short non-self-similar stage (see Fig. 2). The values β_T and β_W are substantially different,

$$\beta_T = 0.93 \pm 0.02; \quad \beta_W = 0.68 \pm 0.02.$$

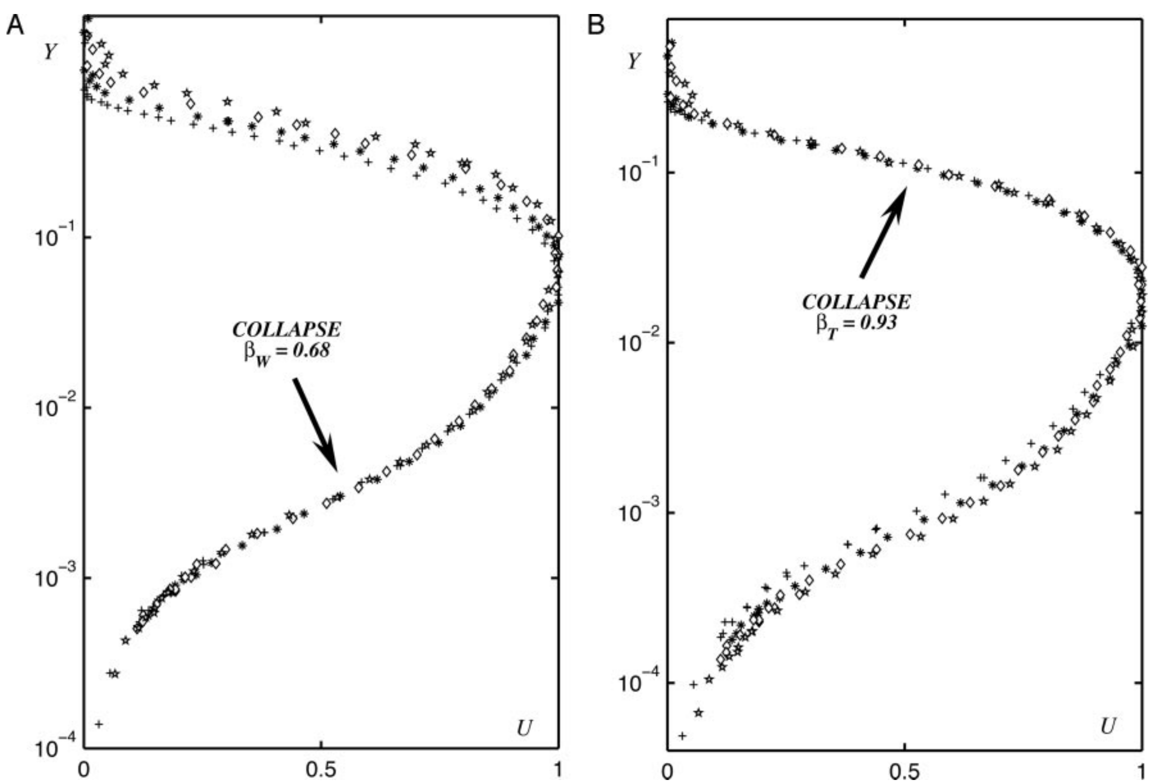

Fig. 3. Collapse of the experimental curves $y/d^{1-\beta_T}x^{\beta_T}$, u/u_{max} (*A*) and $y/d^{1-\beta_W}x^{\beta_W}$, u/u_{max} (*B*) to different scaling laws; +, x/d = 40; ✻, x/d = 70; ◇, x/d = 100; ⋆, x/d = 150.

Having established this fact we processed the velocity distributions from the Classic Database in the coordinates $Y = y/d^{1-\beta_T}x^{\beta_T}$, $U = u/u_{\max}$ (Fig. 3*A*), and $Y = y/d^{1-\beta_W}x^{\beta_W}$, $U = u/u_{\max}$ (Fig. 3*B*).

The result of the processing is instructive: a clear collapse to a single curve in the top part and no collapse in the wall part in the first case, and vice versa in the second case. In both cases, the collapse extended to the values of $u/u_{\max}$ close to one.

Thus, the analysis of the data of Karlsson *et al.* (1) allows us to suggest the following hypotheses concerning the structure of wall jet flow.

First Hypothesis. The flow region consists of three layers (Fig. 1*B*).

1. Top layer, the region around and above the upper line $y^T_{1/2}$ where the mean velocity is equal to one-half of the maximum velocity.
2. Wall layer, the region around and below the lower line $y^W_{1/2}$ where the average velocity is equal to one half of the maximum velocity.
3. Intermediate layer, the region between the top and wall layers where the velocity is close to maximum.

Second Hypothesis. At large Reynolds numbers in the top and wall layers, the flow has the property of incomplete similarity (read about this concept in detail in ref. 13), so that all dimensionless quantities can be represented in the form.

$$\Phi(\Pi_1, \Pi_2, Re) = \Pi_2^{\alpha}\Phi_1\left(\frac{\Pi_1}{\Pi_2^{\beta}}, Re\right), \quad [4]$$

where α and β are Reynolds-number-dependent powers different for the top and wall layers; in the third, intermediate layer the mean velocity is close to the maximum.

We emphasize that these statements are hypotheses and not yet established facts because they are based on a restricted amount of experimental data. In particular, we cannot now say anything about the Reynolds number dependence of the powers. It can be assumed, by analogy with pipes and boundary layers, that $\beta = 1 - \text{Const}/\ln Re$; if this assumption is so, then at very large Reynolds numbers complete similarity will be established.

The relation **4** is in fact a more complicated form of an incomplete similarity relation than the one that we met previously in flows in pipes and boundary layers (14, 15)

$$\Phi = \Pi_1^{\alpha(Re)}C(Re), \quad [5]$$

where $\Pi_1 = y/\delta$, $\delta = \nu/u_*$ is the viscous length scale and u_* is the friction velocity. In the original dimensional variables, the similarity law **4** is represented in the form

$$\Phi = \left(\frac{x}{d}\right)^{\alpha}\Phi_1\left(\frac{y}{d^{1-\beta}x^{\beta}}, Re\right), \quad [6]$$

so that every kinematic property of wall jet flow can be represented as

$$z = \left(\frac{J}{\rho}\right)^{p} d^{q-\alpha}x^{\alpha}\Phi_2\left(\frac{y}{d^{1-\beta}x^{\beta}}, Re\right). \quad [7]$$

Here p and q are Reynolds-number-independent quantities easily obtained by dimensional analysis because the dimension of every kinematic property z can be represented as a product of dimensions of J/ρ, $[J/\rho] = L^3T^{-2}$, and $[d] = L$. In particular,

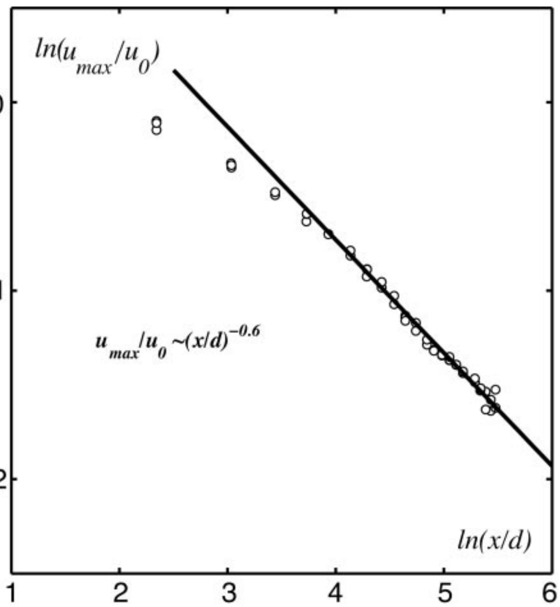

Fig. 4. Experimental data reveal incomplete similarity: $u_{\max}(x)$ is proportional to $x^{-0.6}$ and not $x^{-0.5}$.

the mean velocity distribution $\bar{u}(y)$ in both the top and wall layers can be represented as

$$\bar{u}(y) = \left(\frac{J}{\rho d}\right)^{\frac{1}{2}}\left(\frac{x}{d}\right)^{\alpha}\psi_{\bar{u}}\left(\frac{y}{d^{1-\beta}x^{\beta}}, Re\right), \quad [8]$$

and the relations for the top and wall lines corresponding to the values of the velocity equal to $\frac{1}{2}u_{\max}$ are represented by Eq. **3**.

An important point concerns earlier assumptions of complete similarity, i.e., the possibility of neglecting d. This hypothesis was proposed by many authors starting with Prandtl (2) and Tollmien (3) in the mid-1920s and repeated in various textbooks, in particular by Landau and Lifshitz (16) (for free jets, mixing layers and wakes; wall jets were not considered widely at the time this famous book was composed). According to our processing of the experimental data, this assumption is incorrect and must be abandoned. Under the assumption of complete similarity, we should have $\alpha = -1/2$, $\beta = 1$. Processing of the same high-quality experimental data of Karlsson *et al.* showed (see Fig. 4) that $\alpha = -0.6$, and this fact is an additional argument in favor of incomplete similarity. More generally, when faced with multiscale phenomena, researchers should be cautious in neglecting small parameters.

Here a very important paper by Kotsovinos (17) concerning free jets should be mentioned. He noticed the lack of linear growth of $y_{1/2}$ (x) and proposed a nonlinear relation for this quantity. However, he did not relate it to the influence of the width of the slot d. Processing the data of Kotsovinos has given to us the scaling relation **3** with a value $\beta = 1.1$, differing in a statistically significant way from $\beta = 1$, which also reveals that we are in the presence of incomplete similarity. We direct the reader to a remarkable source of information about turbulent jets: the comprehensive treatises (18, 19) by G. N. Abramovich and colleagues.

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